Name ______________________

Class __________

# YEAR 11

## TERM 3 ASSESSMENT

## 2020

# MATHEMATICS EXTENSION

| | |
|---|---|
| • **General Instructions** | • **Reading time – 5 minutes**<br>• **Working time – 2 hours**<br>• **Write using black pen**<br>• **Calculators approved by NESA may be used**<br>• **A separate reference sheet is provided**<br>• **For questions in Section II, show relevant mathematical reasoning and/or calculations** |
| • **Total marks: 85** | **Section I – 5 marks**<br>• **Attempt Questions 1–5**<br>• **Allow about 7 minutes for this section**<br><br>**Section II – 80 marks**<br>• **Attempt Questions 6-9**<br>• **Allow about 1 hours and 53 minutes for this section** |

**The answers to all questions are to be returned in separate *stapled* bundles, clearly marked with Question 6, Question 7, etc., with your student number.**

**Section 1** **Total marks – 10**

**All questions are of equal value**

Use the multiple-choice answer sheet for Questions 1–10.

1. For what positive values of $n$ does the expansion of $\left(x^2 - \frac{2}{x^3}\right)^n$ have a constant term?

(A) Odd values of $n$

(B) Even values of $n$

(C) Multiples of 3

(D) Multiples of 5

2. Given, $a > 0$, the area of the region bounded by the graphs of $y = \sqrt{2ax - x^2}$ and $y = |x - a| - a$ is equal to:

(A) $\frac{a^2(\pi - 2)}{2}$

(B) $a^2(\pi + 1)$

(C) $\frac{a^2(\pi + 2)}{2}$

(D) $a^2(\pi - 1)$

3. A function is given by the parametric equations: $x = t + \frac{1}{t}$ , $y = t^3 + \frac{1}{t^3}$

Its Cartesian equation is

(A) $y = x^3 - 3x$

(B) $y = x^3 + 3x$

(C) $y = x^2 - x$

(D) $y = x^3$

4. A possible set of parametric equations for the curve $4x^2 + y^2 = 24x - 2y - 33$ is

(A) $x = 3 - \sin(t)\ \ y = 1 + 2\cos(t)$

(B) $x = 3 + \sin(t)\ \ y = -1 + 2\cos(t)$

(C) $x = 2 + \sin(t)\ \ y = 1 + 2\cos(t)$

(D) $x = -3 + \sin(t)\ \ y = 1 + 2\cos(t)$

5. Consider the function: $f(x) = \dfrac{1}{1+e^x} - \dfrac{1}{2}$

Which of the following statements is FALSE?

(A) The graph of the function has two asymptotes

(B) The inverse of the function is also a function

(C) $f(x)$ is an odd function

(D) The graph of the function has an oblique asymptote

**END OF SECTION 1**

**Section 2 Total marks – 80**

**Question 6 (20 marks). Use a SEPARATE writing paper**

(a) Solve the equation: $\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}=\sin^{-1}x$ showing all steps in your calculations. **3**

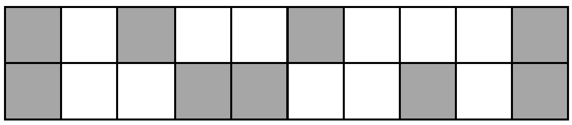
(b) Sketch the graph of the function $y=\frac{1}{2}\cos^{-1}2x$, showing all intercepts. **2**

(c) Consider the equation: $\tan x\tan 2x=c$ where $c$ is a constant,

(i) Show that for the equation to have a solution $c$ must satisfy: $\frac{c}{c+2}\geq 0$ **3**

(ii) Hence, find the values of $c$ for which the equation has a solution. **2**

(d) Two rows of 10 square tiles are to be shaded so that four tiles in the first row and five tiles in the second row are shaded.
Here is an example:



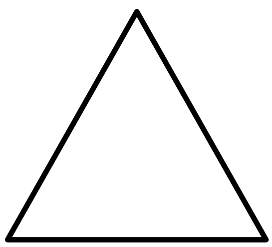
Calculate the number of ways this can be done

(i) with no restrictions **1**

(ii) so that no shaded tile in the top row is directly above a shaded tile in the bottom row. **2**

(iii) so that exactly two shaded tiles in the top row are directly above a shaded tile in the bottom row. **2**

(e) Show that if five points are selected from the interior of an equilateral of side length 2 cm then there are at least two points which are no more than 1cm apart. **2**



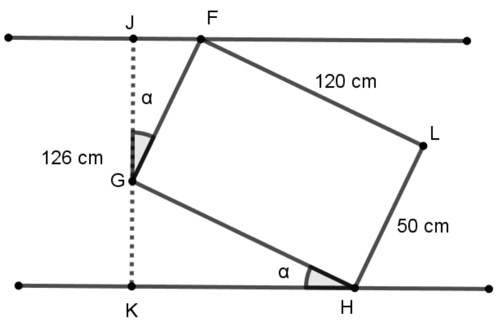
(f) Find the equation of the normal to the curve $y=\frac{3}{\sqrt{2x-1}}$ at the point where $x=1$. **3**

**Question 7 (20 marks). Use a SEPARATE writing paper**

(a) (i) Prove the trigonometric identity $\sin^2\alpha - \sin^2\beta \equiv \sin(\alpha+\beta)\sin(\alpha-\beta)$. **2**

(ii) Hence, or otherwise, solve for $\sin\left(x+\frac{\pi}{6}\right)\sin\left(x-\frac{\pi}{6}\right)=0.25$ in the domain $0 \le x \le 2\pi$. **2**

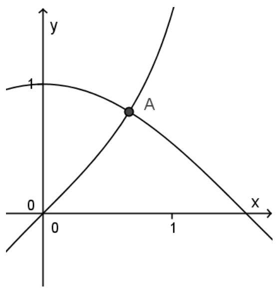
(b) The rectangle *FGHL* touches two parallel lines, 126 cm apart, at opposite vertices. The dimensions of the rectangle are 50 cm and 120 cm, as shown.
Let angle *GHK* be $\alpha$.




(i) Show that $60\sin\alpha + 25\cos\alpha = 63$ (you may assume $\measuredangle JGF = \measuredangle GHK$). **1**

(ii) If $\tan\left(\frac{\alpha}{2}\right)=t$, show that $44t^2 - 60t + 19 = 0$. **2**

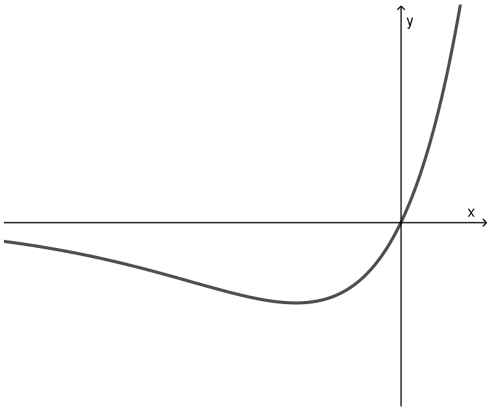
(c) The graphs of $y = \cos x$ and $y = \tan x$ intersect at the point *A*, as shown.




(i) By using suitable trigonometric identities, show that the $x$ coordinate of the point *A* satisfies the equation: $\sin^2 x + \sin x - 1 = 0$. **2**

(ii) Hence, show that at the point *A*, $x = sin^{-1}\left(\frac{-1+\sqrt{5}}{2}\right)$. **2**

(d) The following curve is the graph of the function $f(x) = xe^{x}$




It can be shown that $f(x)$ has one stationary point at $\left(-1, -e^{-1}\right)$
DO NOT PROVE THIS

(i) Draw the graph of $y = |f(-x)|$ indicating any stationary points and asymptotes. **2**

(ii) On the same set of axes, sketch the graph of $y = \sqrt{|f(-x)|}$ indicating any stationary points and asymptotes. **3**

(e) (i) Show that $(1+x)^{m}\left(1-\dfrac{1}{x}\right)^{m} = \left(x-\dfrac{1}{x}\right)^{m}$. **2**

(ii) By considering the term(s) independent of $x$ in the expansion of (i), show that **2**

$$\binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \ldots + \binom{2n}{2n}^2 = (-1)^n \binom{2n}{n}$$

**Question 8 (20 marks). Use a SEPARATE writing paper**

(a) (i) Prove the trigonometric identity: $\sin^4\theta + \cos^4\theta \equiv \dfrac{3+\cos 4\theta}{4}$. **2**

(ii) Hence, or otherwise, solve $\sin^4 x + \cos^4 x = \dfrac{1}{2}$ in the domain $0 \le x \le \pi$. **2**

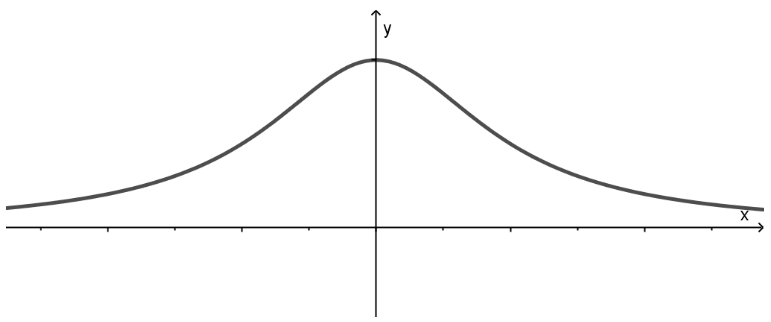
(b) (i) Sketch the graph of $f(x) = \sqrt{x+4} + 1$, stating the domain and range of $f(x)$. **2**

(ii) Find $f^{-1}(x)$, the inverse function of $f(x)$, stating its domain and range. **2**

(iii) Sketch the graph of $f^{-1}(x)$ on the same axes. **2**

(iv) Solve the equation: $f(x) = f^{-1}(x)$ presenting your answer in exact form. **2**

(c) Let $f(x) = \dfrac{1}{x^2+1}$ The graph of $y = f(x)$ is given below.




(i) Show that $f(x) + f\left(\dfrac{1}{x}\right) = 1$ for all values of *x*. **2**

(ii) Hence, or otherwise, sketch the graph of $y = f\left(\dfrac{1}{x}\right)$ noting any asymptotes and intercepts with the axes. **2**

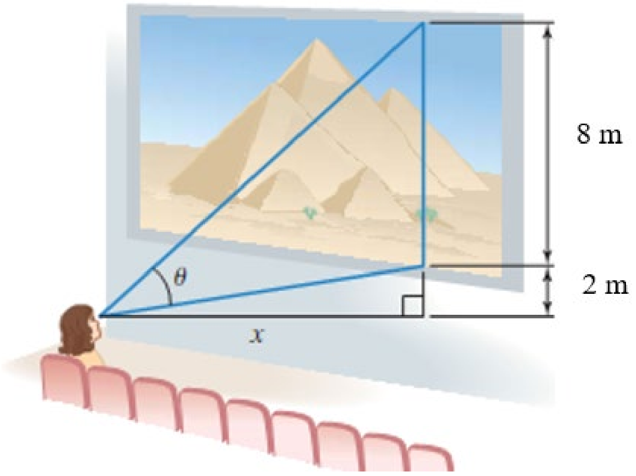
(d) Ten people arrive to eat at a restaurant. The only available seating are two circular tables: one that seats six people, and another that seats four.

(i) Using these tables, how many different seating arrangements are there for the ten people? **2**

(ii) Anna and Tom are in the group. What is the probability that they will be seated at the same table? **2**

**Question 9 (20 marks). Use a SEPARATE writing paper**

(a) Sybil goes to *Giant Cinema* to see a film. The cinema has an 8-metre high screen located 2 metres above her eye level.

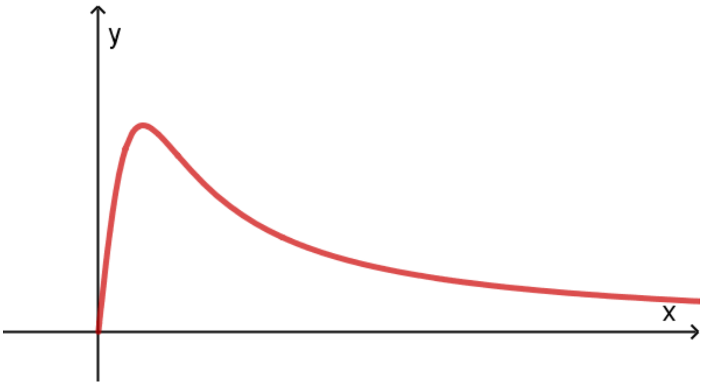



(i) Show that if Sybil sits $x$ metres back from the screen and her viewing angle is $\theta$, then: $\theta = \tan^{-1}\left(\frac{10}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)$. **1**

(ii) Show that $\theta = \tan^{-1}\left(\frac{8x}{x^2+20}\right)$. **2**

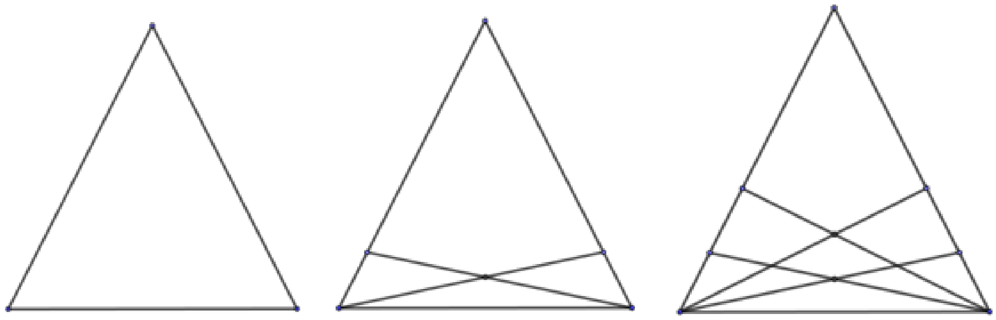
(iii) Consider the curve $y = \frac{8x}{x^2+20}$ and the horizontal line $y = k$. Show that for the line to intersect the curve, $k$ must satisfy $-\frac{2}{\sqrt{5}} \le k \le \frac{2}{\sqrt{5}}$. **2**

(iv) The graph of the curve $y = \frac{8x}{x^2+20}$ in the domain $x \ge 0$ is given:




Using this graph and part (iii), and **without** using calculus, show that in *Giant Cinema*, Sybil's viewing angle, $\theta$, is maximized if she sits $2\sqrt{5}$ metres away from the screen. **2**

(b) Parminder is generating tirangles from a given triangle, $T_1$. He takes the triangle and draws a ray from each corner of the base angles, as shown, and notes the number of triangles this creates in $T_2$. He then creates a third triangle, $T_3$, by drawing another ray from each base angle, and counts the number of triangles now. He continues doing this, creating a series: $T_1$, $T_2$, $T_3$,…

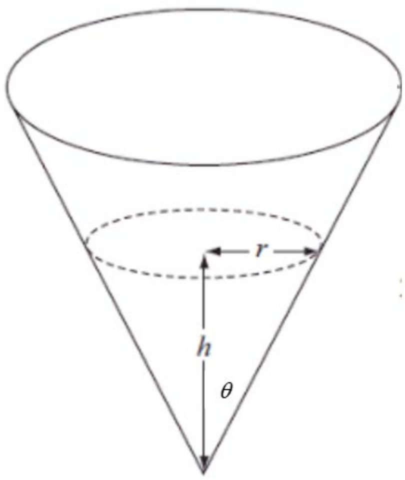


**$T_1 = 1$** **$T_2 = 8$** **$T_3 = 27$** **… $T_n =$**

(i) Explain why $T_n = \binom{2n+1}{3} - 2\binom{n+1}{3}$, $\quad n \geq 2$ **2**

(ii) Hence, prove that $T_n = n^3$. **2**

(c) A water tank has the shape of a hollow inverted right circular cone with its axis vertical and vertex lowest. Its semi vertical angle is $\theta = \tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10 m is (in meter/min) **3**




**QUESTION 9 CONTINUES NEXT PAGE**

(d) The population size, $P$, of a Tasmanian devil colony in a certain park at time $t$ years is montiored. $P$ satisfies the model: $\frac{dP}{dt} = -k(P - A)$ for some constants, $A$ and $k$.

(i) Verify that $P = A + Be^{-kt}$, $B$ constant, satisfies this equation. **1**

(ii) Initially there were $n$ Tasmanian devils in the park and a year later $m$ are left.
Show that $P = A + B\left(\frac{m-A}{n-A}\right)^t$ **3**

(iii) The population size of koalas in a different park, satisfies the mathematical model: $\frac{dP}{dt} = -k(P - 100)$. Initially there were 650 koalas in the par, but a year later only 500 were counted. Calculate when the number of koalas in the park will fall below 150 for the first time. **2**

You may use results from previous parts of this question.

**END OF EXAMINATION**

| Question | MC | 6 | | 7 | | 8 | | 9 | | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Trig Functions | | **a,c** | /8 | **a,d,f** | **/11** | **d** | **/4** | **c** | **/7** | **/30** |
| Functions | | | | **b** | /5 | **a,c** | /12 | | | **/17** |
| Combinatorics | | **d,e** | **/7** | **e** | /4 | **b** | /4 | **a** | **/4** | **/19** |
| Calculus | | **b** | **/3** | | | | | **b,d** | **/9** | **/12** |
| **MC** | | | | | | | | | | **/5** |
| Total | **/5** | | **/18** | | **/20** | | **/20** | | **/20** | **/83** |

**MATHEMATICS Extension 1, Question 6**

| Suggested Solutions | Marks Awarded | Marker's Comments |
|---|---|---|
| (a) $$\sin^{-1} x = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$$ so | | Most students used right-angled triangle to find inverse trigonometrics, which was fine. |
| $$x = \sin\left(\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}\right)$$ | | Alternative method presented here. |
| $$= \sin\left(\sin^{-1}\frac{5}{13}\right)\cos\left(\cos^{-1}\frac{3}{5}\right) + \cos\left(\sin^{-1}\frac{5}{13}\right)\sin\left(\cos^{-1}\frac{3}{5}\right)$$ | 1 | The only significant problem was using a calculator to evaluate: any question like this assumes you work through exactly. |
| $$= \frac{5}{13}\cdot\frac{3}{5} + \sqrt{1-\sin^2\left(\sin^{-1}\frac{5}{13}\right)}\sqrt{1-\cos^2\left(\cos^{-1}\frac{3}{5}\right)}$$ | | First mark for unpacking the compound angle, second mark for evaluating the correct inverse trigonometric values, third mark for correct final answer. |
| $$= \frac{3}{13} + \sqrt{1-\left(\frac{5}{13}\right)^2}\sqrt{1-\left(\frac{3}{5}\right)^2}$$ | 1 | |
| $$= \frac{63}{65}$$ | 1 | |
| (b) $$y = \frac{3}{\sqrt{2x-1}} \to \frac{dy}{dx} = -\frac{3}{\sqrt{(2x-1)^3}}$$ | 1 | Common errors:<br>• not using chain rule to take derivative of radicand, $(2x-1)' = 2$;<br>• finding tangent gradient and finding tangent line;<br>• missing negative on derivative. |
| Gradient at $x = 1$ given by $\frac{dy}{dx}\vert_{x=1} = -3$, so gradient of the normal is $\frac{1}{3}$. Hence the normal at $x = 1$ is: $$y - y(1) = \frac{1}{3}(x-1)$$ $$y - 3 = \frac{1}{3}(x-1)$$ | 1 | |
| $$x - 3y + 8 = 0$$ | 1 | One mark for correct derivative, one for correct normal gradient, one mark for correct normal. |

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| (c)<br>(i) Given $\tan x \tan 2x = c$:<br>$$c = \tan x \tan 2x = \tan x \frac{2\tan x}{1-\tan^2 x}$$<br>$$= \frac{2\tan^2 x}{1-\tan^2 x}$$<br>Let $t = \tan x$. Then<br>$$\frac{2t^2}{1-t^2} = c \to 2t^2 = c - ct^2$$<br>$$t^2(2+c) = c$$<br>$$t^2 = \frac{c}{2+c} \geq 0$$<br>since $t^2 \geq 0$. | 1<br><br>1<br><br>1 | There was a lot of confusion in responses and many students used convoluted methods, such as<br>$$c = \frac{2\tan^2 x}{1-\tan^2 x}$$<br>so that<br>$$\frac{c}{2+c} = \frac{\left(\frac{2\tan^2 x}{1-\tan^2 x}\right)}{\left(2+\frac{2\tan^2 x}{1-\tan^2 x}\right)}$$<br>$$= \tan^2 x \geq 0$$<br>Technically fine, but a ‘mission’.<br><br>One mark for converting double angle to single, one mark for quadratic form, one mark for explaining why the result must be greater than or equal to 0. |
| (ii) The equation will have a solution for all $c$ satisfying the inequality in (i), so<br>$$\frac{c}{2+c} \times (2+c)^2 \geq 0 \times (2+c)^2 = 0$$<br>$$c(2+c) \geq 0$$<br>which is a concave-up quadratic with intercepts at $c = 0$ and $-2$. Hence the inequality is satisfied for all $c$ where $c < -2$ and $c \geq 0$. Note the strict inequality since if $c = 2$, the ratio in the original inequality (what we’re actually solving) won’t exist.<br><br>So $c \in (-\infty, -2) \cup [0, \infty)$. | 1<br><br>1 | Greatest error here was including $-2$ in the solution.<br><br>If any answer was incorrect but working was correct: 1 mark.<br><br>If one of two answers correct: 1 mark.<br><br>Did not need to express solution using sets. |

| | | |
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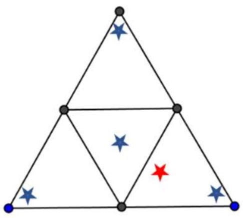
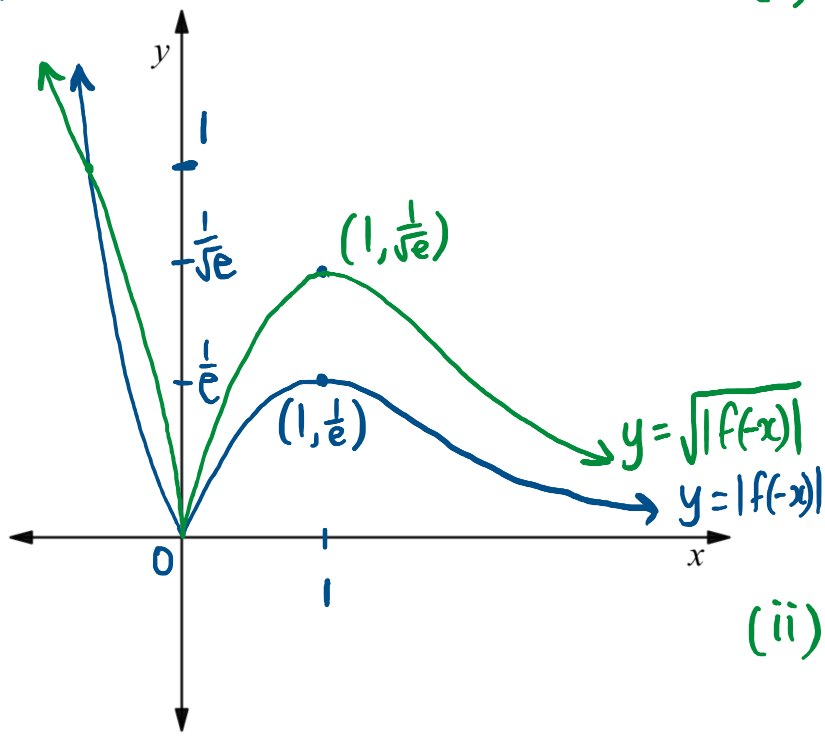
| (d)<br><br><br>(i) There are $\binom{10}{5}$ ways to colour the bottom row, and for each of these ways, $\binom{10}{4}$ ways to colour the top row. Hence there are $\binom{10}{5}\binom{10}{4} = 59920$ ways to colour the rows without restriction. | 1 | Biggest mistakes here:<br>• using permutations instead of combinations;<br>• adding instead of multiplying, as the Fundamental Theorem of Counting would require. |
| (ii) First, there are $\binom{10}{5}$ ways to shade the bottom row. No shading may exist above a shaded tile, so we have only 5 possible tiles to shade. Hence there are $\binom{5}{4}$ ways of shading the top row. There are therefore $\binom{10}{5}\binom{5}{4} = 1260$ ways of completing the problem. | 1<br><br>1 | Question was mishandled by about one third of the candidature.<br><br>In (ii) and (iii), one mark for getting combination correct for one of the rows or appropriate, logical, **relevant** working. |
| (iii) Again, there are $\binom{10}{5}$ ways of shading the bottom row. We colour two tiles above a shaded tile below, and there are $\binom{5}{2}$ ways of doing this. No other tile in the top row having a coloured tile directly beneath it may be used, leaving five tiles to fill using two shadings, and there are $\binom{5}{2}$ ways of doing this. Hence there are $\binom{10}{5}\binom{5}{2}\binom{5}{2} = 25200$ ways of colouring the tiles. | 1<br><br>1 | Second mark, obviously, for complete, correct answer. |
| (e)<br><br>The side lengths of the larger triangle are all 2cm. Select the midpoint of each side and connect them to form four triangles within. Now, by midsegment theorem, each of the lengths joining the midpoints are half the length of, and parallel to, the remaining side. So, we construct four equilateral triangles having side length 1cm.<br><br>The largest possible distance between two points of one small triangle is given by the length of its side: 1 cm. We have four smaller triangles and five points. By the Pigeonhole Principle, we have four pigeonholes (triangles) | 1 | Note: the pigeonhole principle is not the only way this question could be answered.<br><br>If only one mark was awarded, it was due to a lack of justification as to why the distance between two points in a smaller triangle would be no large than 1 cm. Not much was needed, just a simple statement as to why.<br><br>If candidates moved from establishing four smaller triangles and then stated that the fifth point could be no further than 1 cm, or something similar, this was |

| | | |
|---|---|---|
| and five pigeons (points), so at least one triangle must contain at least two points. Since the largest distance between two points is the edge length, the distance between any two points in a single smaller triangle does not exceed 1 cm.<br><br>Hence there'll always be at least two points that are no more than 1 cm apart. | **1** | insufficient for the mark. Although true, the problem is that the candidate knows what needs to be said in order to make the problem work.<br><br>Marks were awarded throughout the cohort based on relative quality.<br><br>Note: although the question talks of the interior of the triangle, there was no penalty if students used the boundary or vertices of the larger triangle. |

| MATHEMATICS Extension 1 : Question 7 | | |
|---|---|---|
| **Suggested Solutions** | **Marks** | **Marker's Comments** |
| Method 1:<br>a) (i) $RHS = \sin(\alpha+\beta)\sin(\alpha-\beta)$<br>$= \frac{1}{2}\left[\cos((\alpha+\beta)-(\alpha-\beta)) - \cos((\alpha+\beta)+(\alpha-\beta))\right]$ | 1 | Using product to sum and simplifying |
| $= \frac{1}{2}\left[\cos 2\beta - \cos 2\alpha\right]$<br>$= \frac{1}{2}\left[1-2\sin^2\beta - (1-2\sin^2\alpha)\right]$<br>$= \frac{1}{2}\left[2\sin^2\alpha - 2\sin^2\beta\right]$<br>$= \sin^2\alpha - \sin^2\beta$<br>$= LHS$ | 1 | Using double angle results and simplifying |
| Method 2:<br>$RHS = \sin(\alpha+\beta)\sin(\alpha-\beta)$<br>$= (\sin\alpha\cos\beta + \cos\alpha\sin\beta)(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$ | 1 | expansion of sum and difference |
| $= \sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta$ difference of two squares<br>$= \sin^2\alpha(1-\sin^2\beta) - (1-\sin^2\alpha)\sin^2\beta$<br>$= \sin^2\alpha - \sin^2\alpha\sin^2\beta - \sin^2\beta + \sin^2\alpha\sin^2\beta$<br>$= \sin^2\alpha - \sin^2\beta$<br>$= LHS$ | 1 | using Pythagorean identities and simplifying |
| (ii) $\sin\left(x+\frac{\pi}{6}\right)\sin\left(x-\frac{\pi}{6}\right) = 0.25$<br>$\sin^2 x - \sin^2\frac{\pi}{6} = 0.25$ using (i)<br>$\sin^2 x - \left(\frac{1}{2}\right)^2 = \frac{1}{4}$<br>$\sin^2 x = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$<br>$\sin x = \pm\sqrt{\frac{1}{2}}\ ,\ 0 \le x \le 2\pi$ | | Mistakes which resulted in:<br>$\sin x = \pm\frac{\sqrt{3}}{2}$<br>max 1 mark for 4 solutions<br>$\sin x = 0$<br>max 1 mark for 3 solutions |
| $\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ | 1 | for 2 solutions in radians |
| | 1 | for another 2 solutions |

## MATHEMATICS Extension 1: Question 7

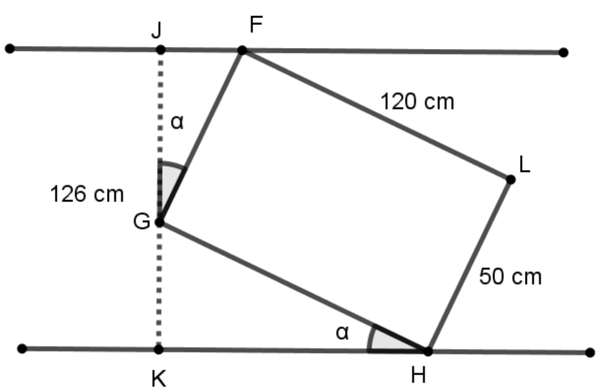
b) (i)




| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| (i) | 1 | stationary point at $(1, \frac{1}{e})$ and graph passing through $(0,0)$ |
| | 1 | shape including sharp point at $(0,0)$. |
| (ii) | 1 | stationary point at $(1, \frac{1}{\sqrt{e}})$ and graph passing through $(0,0)$. |
| Question specified "same axes" $\therefore$ graphs must be drawn on same axes to show point of intersection. | 1 | shape including sharp point at $(0,0)$. |
| | 1 | point of intersection of functions at $y=1$. |

**MATHEMATICS Extension 1: Question 7**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| c) In triangle JGF : $\cos\alpha = \frac{JG}{50} \Rightarrow JG = 50\cos\alpha$ <br> In triangle GKH : $\sin\alpha = \frac{GK}{120} \Rightarrow GK = 120\sin\alpha$ | 1 | must show $\cos\alpha$, $\sin\alpha$ expressions |
| $GK + JG = 126$ <br> $120\sin\alpha + 50\cos\alpha = 126$ <br> $60\sin\alpha + 25\cos\alpha = 63$ <br> Note: cannot assume that G is the midpoint of JK. | 1 | showing substitution into $GK + JG = 126$ |
| (ii) Let $t = \tan\frac{\alpha}{2}$ <br> $\sin\alpha = \frac{2t}{1+t^2}$, $\cos\alpha = \frac{1-t^2}{1+t^2}$ from Reference sheet <br> $60\sin\alpha + 25\cos\alpha = 63$ <br> $\therefore 60\times\frac{2t}{1+t^2} + 25\times\frac{1-t^2}{1+t^2} = 63$ | 1 | correct substitution |
| $60\times 2t + 25(1-t^2) = 63(1+t^2)$ <br> $120t + 25 - 25t^2 = 63 + 63t^2$ <br> $0 = 38 - 120t + 88t^2$ <br> $\therefore 44t^2 - 60t + 19 = 0$ | 1 | Correct expansion and simplification |



MATHEMATICS Extension 1: Question 7

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| d) (i) LHS $= (1+x)^m \left[1-\frac{1}{x}\right]^m$ | | |
| $= \left[(1+x)\left(1-\frac{1}{x}\right)\right]^m$ | 1 | single expression to power of m |
| $= \left[1-\frac{1}{x}+x-1\right]^m$ | 1 | correct expansion and simplification |
| $= \left(x-\frac{1}{x}\right)^m$ = RHS | | |
| (ii) Consider the expansions: $(1+x)^m = \binom{m}{0}+\binom{m}{1}x+\binom{m}{2}x^2+\dots+\binom{m}{m}x^m$ <br> $\left(1-\frac{1}{x}\right)^m = \binom{m}{0}-\binom{m}{1}\frac{1}{x}+\binom{m}{2}\frac{1}{x^2}+\dots+\binom{m}{m}\frac{1}{x^m}(-1)^m$ | | most successful solutions did not use sigma notation to show constant term on LHS |
| $\therefore$ In $(1+x)^m\left(1-\frac{1}{x}\right)^m$ the term independent of $x$ is $\binom{m}{0}^2-\binom{m}{1}^2+\binom{m}{2}^2+\dots+\binom{m}{m}(-1)^m$ | 1 | derivation of constant term on LHS |
| $\left(x-\frac{1}{x}\right)^m = \sum_{r=0}^{m}\binom{m}{r}x^r\left(\frac{-1}{x}\right)^{m-r}$ <br> $= \sum_{r=0}^{m}\binom{m}{r}x^r(x)^{r-m}(-1)^{m-r}$ <br> $= \sum_{r=0}^{m}\binom{m}{r}x^{2r-m}(-1)^{m-r}$ | 1 | derivation of constant term on RHS and equating these with correct use of m, r, n. |
| The term independent of $x$ occurs when $2r-m=0$ ie. $m=2r$ <br> ie. $\binom{2r}{r}(-1)^r$ | | |
| When $m=2r$: <br> $\binom{2r}{0}^2-\binom{2r}{1}^2+\binom{2r}{2}^2-\dots\binom{2r}{2r}^2=\binom{2r}{r}(-1)^r$ <br> ie $\binom{2n}{0}^2-\binom{2n}{1}^2+\binom{2n}{2}^2-\dots\binom{2n}{2n}^2=\binom{2n}{n}(-1)^n$ | | |

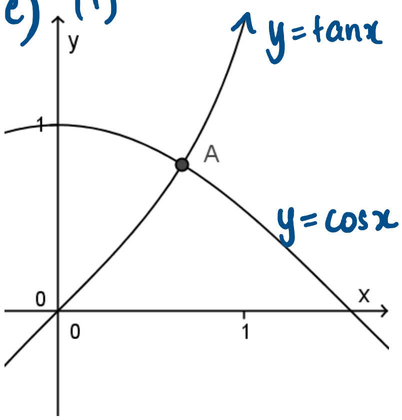
**MATHEMATICS Extension 1: Question 7**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| | | |

## MATHEMATICS Extension 1: Question 7

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| Using sigma notation: | | Students must ensure the correct use of the word 'independent'. |
| $(1+x)^m = \sum_{r=0}^{m} \binom{m}{r} x^r$ | | |
| $(1-\frac{1}{x})^m = \sum_{j=0}^{m} \binom{m}{j}\left(-\frac{1}{x}\right)^j = \sum_{j=0}^{m}\binom{m}{j}(-1)^j x^{-j}$ | | |
| The terms in the expansion of $(1+x)^m(1-\frac{1}{x})^m$ take the form | | First mark was awarded for correctly explaining the derivation of the term independent of $x$ in the expansion of LHS. |
| $\binom{m}{r}x^r\binom{m}{j}(-1)^j x^{-j} = \binom{m}{r}\binom{m}{j}x^{r-j}(-1)^j$ | | |
| For terms independent of $x$, $r=j$ | | |
| ie $\binom{m}{r}\binom{m}{r}(-1)^r = \binom{m}{r}^2(-1)^r$ | | |
| $\therefore$ Term independent of $x$ | | |
| $= \binom{m}{0}^2 + \binom{m}{1}^2(-1) + \binom{m}{2}^2(-1)^2 + \dots + \binom{m}{m}^2(-1)^m$ | 1 | |
| when $m=2n$ | | |
| $= \binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \dots + \binom{2n}{2n}$ | | |
| $(x-\frac{1}{x})^m = \sum_{r=0}^{m}\binom{m}{r}x^r\left(-\frac{1}{x}\right)^{m-r}$ | | Second mark for explaining the term independent of $x$ in the expansion of RHS. |
| $= \sum_{r=0}^{m}\binom{m}{r}x^r(-1)^{m-r}(x)^{r-m}$ | | |
| $= \sum_{r=0}^{m}\binom{m}{r}x^{2r-m}(-1)^{m-r}$ | | |
| For term independent of $x$ $2r=m$ | | |
| ie $\binom{m}{r}(-1)^r$ | | |
| when $m=2n$ $\binom{2n}{n}(-1)^n$ | | |

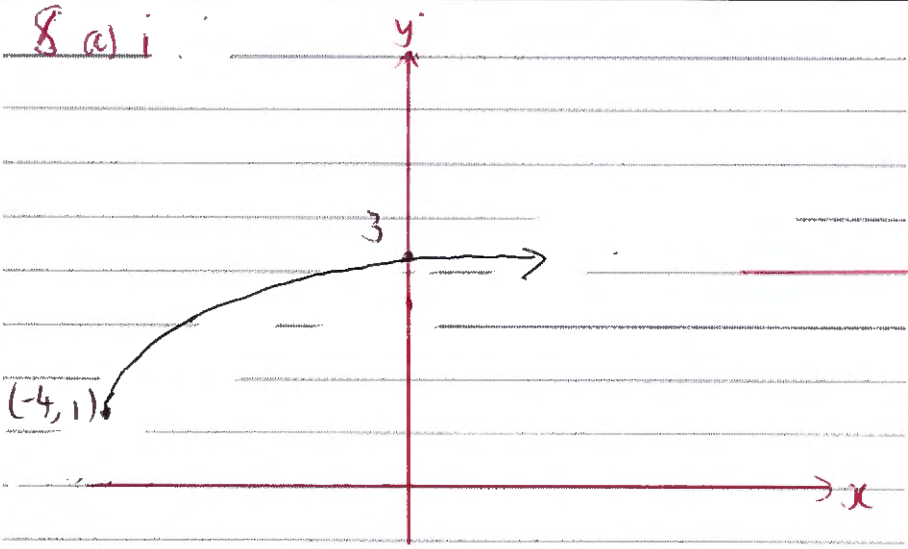
## MATHEMATICS Extension 1: Question 7

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| $\therefore \binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \cdots + \binom{2n}{2n}^2 = \binom{2n}{n}(-1)^n$ | | |
| e) (i)   For intersection: $\tan x = \cos x$ <br> $\dfrac{\sin x}{\cos x} = \cos x$ | 1 | Write $\tan x$ in terms of $\sin x$ and $\cos x$ |
| $\sin x = \cos^2 x$ <br> $= 1 - \sin^2 x$ <br> $\therefore \sin^2 x + \sin x - 1 = 0$ | 1 | Write $\cos^2 x$ as $1-\sin^2 x$ and simplify |
| (ii) Using the quadratic formula: <br> $\sin x = \dfrac{-1 \pm \sqrt{1 - 4\times1\times-1}}{2}$ <br> $= \dfrac{-1 \pm \sqrt{5}}{2}$ | 1 | Using quadratic formula to find $\sin x$ |
| Either $-1 \le \sin x \le 1$ <br> OR $x > 0$ since A is in first quadrant. <br> $\therefore \sin x = \dfrac{-1+\sqrt{5}}{2}$ <br> $\therefore x = \sin^{-1} \dfrac{-1+\sqrt{5}}{2}$ | 1 | explaining the choice of $\dfrac{-1+\sqrt{5}}{2}$ and finding $x$. |

**MATHEMATICS Extension 1 : Question 8**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|

8 (a) i




① for everything right

Domain: $\{x : x \in \mathbb{R},\ x \geq -4\}$ or $[-4, \infty)$

Range: $\{y : y \in \mathbb{R},\ y \geq 1\}$ or $[1, \infty)$ ①

ii let $y = \sqrt{x+4} + 1$

Inverse is given by $x = \sqrt{y+4} + 1$

$$\therefore \sqrt{y+4} = x - 1$$
$$y + 4 = (x-1)^2$$
$$y = (x-1)^2 - 4$$ ①

Domain: $\{x : x \in \mathbb{R},\ x \geq 1\}$ or $[1, \infty)$

Range: $\{y : y \in \mathbb{R},\ y \geq -4\}$ or $[-4, \infty)$ ①

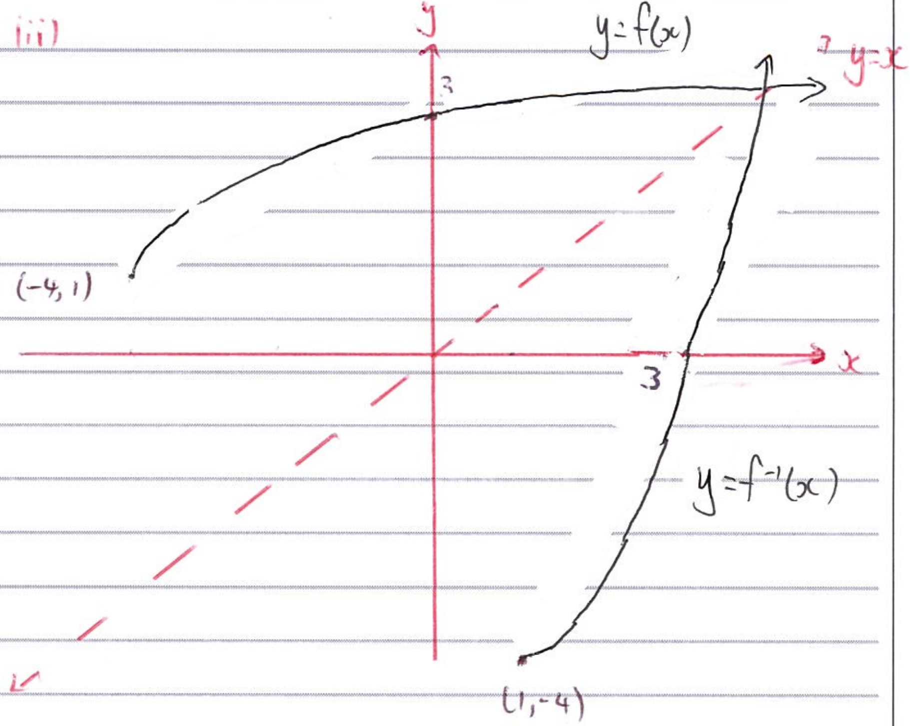
$\therefore f^{-1}(x) = (x-1)^2 - 4$ for $x \geq 1$ only.

or

$$\left.\begin{aligned} f^{-1}(x) &= x^2 - 2x - 3 \\ &= (x-3)(x+1) \end{aligned}\right\} \text{ for } x \geq 1 \text{ only}$$

**MATHEMATICS Extension 1 : Question……..**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| iii) | | Inverse Graph |
| | | * Full marks for everything right |
| | | * 1 mark deducted per bits of information missing/incorrect. |




iv) Since the 2 graphs must intersect at $y = x$,

solve $\sqrt{x+4} + 1 = x$

$\sqrt{x+4} = x - 1$

$x + 4 = x^2 - 2x + 1$

$x^2 - 3x - 3 = 0$ ——— (1)

$\therefore x = \dfrac{3 \pm \sqrt{9+12}}{2}$

$= \dfrac{3 \pm \sqrt{21}}{2}$

Since $x \geq 1$ (Domain of $f^{-1}(x)$)

$x = \dfrac{3 + \sqrt{21}}{2}$ only. ——— (1)

| MATHEMATICS Extension 1 : Question…….. | | |
|---|---|---|
| **Suggested Solutions** | **Marks** | **Marker's Comments** |

b) i $^{10}C_6 \times 5! \times 3! = 151200$

Choose 6 people to be in the table of 6

Arranging 6 people around a table

Arranging 4 people around a table

* Full marks for correct answer

* 1/2 if logic is evident and CLEARLY explained.

ii) P(same table) = P(table of 6) + P(table of 4)

If table of 6 – $^8C_4$ (neglect how tables are arranged)

If table of 4 – $^8C_2$ (neglect how tables are arranged)

$$\therefore P(\text{same table}) = \frac{^8C_4 + {}^8C_2}{^{10}C_4}$$

$$= \frac{7}{15}$$

or

If table of 6 – $^8C_4 \times 5! \times 3! = 50400$

If table of 4 – $^8C_2 \times 5! \times 3! = 20160$

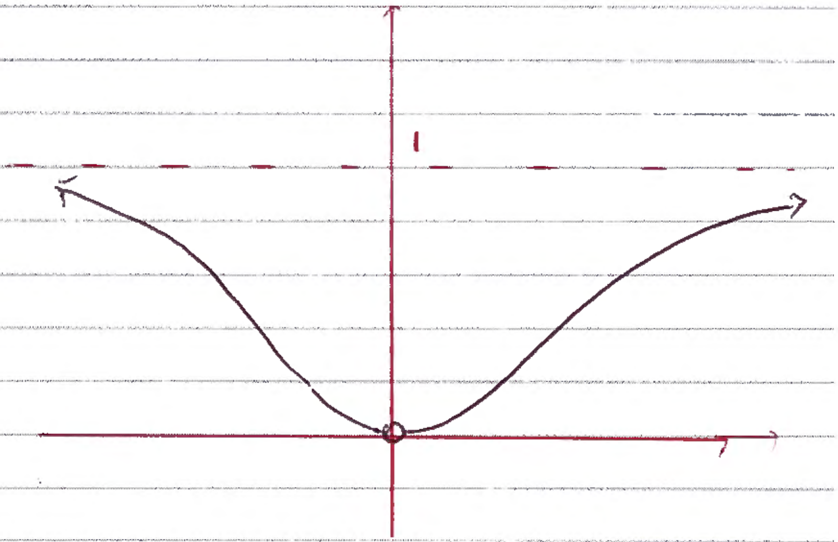
$$\therefore P(E) = \frac{50400 + 20160}{151200}$$

$$= \frac{7}{15}$$

* Full marks for correct answer

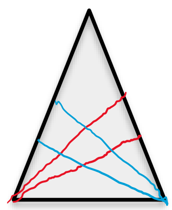
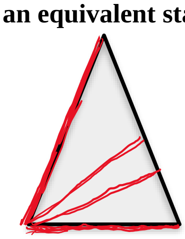
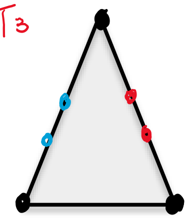
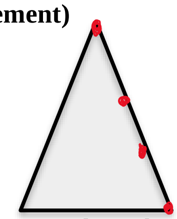
* 1/2 if logic is evident and CLEARLY explained.

**MATHEMATICS Extension 1 : Question........**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| c) i $f\left(\frac{1}{x}\right) = \dfrac{1}{\left(\frac{1}{x}\right)^2 + 1} = \dfrac{x^2}{1+x^2} \quad (x \neq 0)$ | | 1 |
| $\therefore f(x) + f\left(\frac{1}{x}\right) = \dfrac{1}{x^2+1} + \dfrac{x^2}{1+x^2}$ | | |
| $= \dfrac{1+x^2}{1+x^2}$ | | 1 |
| $= 1$ | | |
| ii  | | * Full marks for everything right<br>* 1 mark deducted per detail missing. |

**MATHEMATICS Extension 1 : Question........**

| Suggested Solutions | Marks | Marker's Comments |
|---|---|---|
| d) i $LHS = \sin^4\theta + \cos^4\theta$ | | |
| $= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$ | | |
| $= 1^2 - \frac{1}{2} \times 4\sin^2\theta\cos^2\theta$ | | |
| $= 1 - \frac{1}{2}(2\sin\theta\cos\theta)^2$ | | |
| $= 1 - \frac{\sin^2 2\theta}{2}$ | | 1 |
| $= 1 - \frac{1}{4}(1-\cos 4\theta)$ (Since $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ $= 1 - 2\sin^2 2\theta$ $\therefore \sin^2 2\theta = \frac{1-\cos 4\theta}{2}$) | | |
| $= \frac{4 - (1-\cos 4\theta)}{4}$ | | |
| $= \frac{3+\cos 4\theta}{4}$ | | 1 |
| ii) $\therefore \frac{3+\cos 4x}{4} = \frac{1}{2}$ (from i) | | |
| $3 + \cos 4x = 2$ $\quad 0 \le x \le \pi$ | | |
| $\cos 4x = -1$ $\quad 0 \le 4x \le 4\pi$ | | |
| $\therefore 4x = \pi, 3\pi$ | 1 | |
| $\therefore x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ | 1 | |

| MATHEMATICS Extension 1 Question 9 | | | | |
|---|---|---|---|---|
| **Q** | **Suggested Solutions** | | **Mark** | **Comments** |
| **a i)** | **Method 1: Line segments**<br>$T_3$<br><br>$2n+1$ is the total number of line segments ( 3 + 2(n-1) = 2n + 1)<br><br>$\binom{2n+1}{3}$: choosing 3 line segments from $2n+1$ available line segments to form a triangle<br><br>or<br><br>choosing 3 line segments from $2n+1$ available line segments to find potential number of triangles, **but not all of them are triangles (or an equivalent statement)**<br><br>$n+1$ is total number of line segments from one base vertex<br><br>$\binom{n+1}{3}$: choose 3 line segments from $n+1$ line segments from the same base vertex (or concurrent line segments). These combinations do not form triangles<br><br>$2\binom{n+1}{3}$: two base vertices → multiply by 2 | **Method 2: Points**<br>$T_3$<br><br>$2n+1$ is the total number of points<br><br>$\binom{2n+1}{3}$: choosing 3 points from $2n+1$ available points to form a triangle<br><br>or<br><br>choosing 3 points from $2n+1$ available points to find potential number of triangles, **but not all of them are triangles (or an equivalent statement)**<br><br>$n+1$ is total number of points from each side<br><br>$\binom{n+1}{3}$: choose 3 points from $n+1$ points on the same side (or collinear points from each side). These combinations do not form triangles<br><br>$2\binom{n+1}{3}$: two base vertices → multiply by 2 | 2<br><br>1<br><br>1 | **Common mistakes:**<br>1. $\binom{2n+1}{3}$ refers to all possible triangles (without pointing out that some combinations are not triangles)<br><br>2. $\binom{n+1}{3}$ refers to number of overlapping triangles<br><br>3. Do not explain why multiply by 2<br><br>4. The statement is too general, without much details to support<br><br>5. Leave the symbol C in the answers without explaining its meaning<br><br>6. Prove by using mathematical induction<br><br>7. Prove by using combinatorics (Qa ii) ) (The questions ask to explain) |

| **MATHEMATICS Extension 1 Question 9** | | | |
|---|---|---|---|
| **Q** | **Suggested Solutions** | **Mark** | **Comments** |
| **a ii)** | $T_n = \frac{(2n+1)!}{(2n-2)! \times 3!} - \frac{2(n+1)!}{(n-2)! \times 3!}$<br>$= \frac{(2n-1)(2n)(2n+1)}{6} - \frac{2(n+1)(n)(n-1)}{6}$<br>$= \frac{2n}{6}\left[(2n-1)(2n+1) - (n+1)(n-1)\right]$<br>$= \frac{n}{3}\left(4n^2-1-n^2+1\right)$<br>$= \frac{n}{3} \times 3n^2 = n^3.$<br><br>**First mark**: simplified expressions after removing factorial signs<br>**Second mark:** correct proof involving combine two fractions and combine like terms | **2**<br><br>1<br><br>1 | **Common mistakes:**<br><br>1. Combine two fractions before simplifying the terms, which does not lead to the correct answer<br><br>2. Too little working out<br><br>3. Incorrect calculation in the middle process but still write down the proof statement<br><br>**Long Methods:**<br>After simplifying the terms, multiply all the brackets, cancel like terms, and then simplify again<br><br>**Suggestion:**<br>Remove complicated terms involving factorial signs, factor out common terms and keep it in the front, simplify the rest of the terms |

## MATHEMATICS Extension 1 Question 9

| Q | Suggested Solutions | Mark | Comments |
|---|---|---|---|
| b | (see below) | 3 | Common mistakes: (see below) |

**Q: b**

### Method One (dv/dh)

**First mark**

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$

$= \frac{1}{12}\pi h^3$

$\frac{dV}{dh} = \frac{1}{4}\pi h^2$

**Second mark**

$\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$

$= 5 \div \frac{\pi h^2}{4}$

$= \frac{20}{\pi h^2}.$

**Third mark**

when $h = 10$, $\frac{dh}{dt} = \frac{20}{100\pi} = \frac{1}{5\pi}$

$\doteqdot 0.06$ (2d.p.)

### Method Two + Three (dv/dr)

**First mark** — Mark: 1

$h = 2r$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (2r)$

$= \frac{2}{3}\pi r^3$

$\frac{dV}{dr} = 2\pi r^2$

$\frac{dr}{dt} = \frac{dV}{dt} \div \frac{dV}{dr}$

$= 5 \div 2\pi r^2$

$= \frac{5}{2\pi r^2}.$

**Second mark (version 1)** — Mark: 1

$\frac{dh}{dt} = \frac{dh}{dr} \times \frac{dr}{dt}$ $\left[h = 2r,\ \frac{dh}{dr} = 2\right]$

$= 2 \times \frac{5}{2\pi r^2}$

$= \frac{5}{\pi r^2} = \frac{5}{\pi\left(\frac{h}{2}\right)^2} = \frac{20}{\pi h^2}$

**Third mark (version 1)** — Mark: 1

when $h = 10$, $\frac{dh}{dt} = \frac{20}{100\pi} = \frac{1}{5\pi}$

$\doteqdot 0.06$ (2d.p.)

**Second mark (version 2)**

$\frac{dh}{dt} = \frac{dh}{dr} \times \frac{dr}{dt}$ $\left[h = 2r,\ \frac{dh}{dr} = 2\right]$

$= 2 \times \frac{5}{2\pi r^2}$

$= \frac{5}{\pi r^2}$

### Comments

**Common mistakes:**

1. Both $h$ and $r$ are variables. Cannot treat one of them as a constant in the process of differentiation

$$V = \frac{1}{3}\pi r^2 h$$

Wrong

$$\frac{dV}{dr} = \frac{2}{3}\pi r h$$

$$\frac{dV}{dh} = \frac{1}{3}\pi r^2$$

Correct

$$\frac{dV}{dh} = \frac{1}{3}\pi\left(r^2 + h \times 2r\frac{dr}{dh}\right)$$
$$= \frac{1}{3}\pi\left(r^2 + h \times 2r \times \frac{1}{2}\right)$$
$$= \frac{1}{3}\pi(r^2 + hr)$$

2. Using the chain rule wrong

$$\frac{dh}{dt} = \frac{\pi h^2}{4} \times 5$$
$$\frac{dh}{dt} = \frac{\pi h^2}{4} \div 5$$

3. Treat $\frac{dr}{dt}$ as $\frac{dh}{dt}$

4. Sub in the value of $h$ or $r$ before taking derivaties. (If students sub values without using chain rule, no ECF marks for value substituting step)

5. Mistakes in the volume formula (no $\frac{1}{3}$ or $\pi$)

6. Calculation mistakes!

7. Basic derivative mistakes!

| | | | | |
|---|---|---|---|---|
| | | **Third mark (version 2)**<br><br>when $h=10$, $r=5$<br>$\frac{dh}{dr} = \frac{5}{\pi(5)^2} = \frac{1}{5\pi}$. | | **Suggestions:**<br><br>1. Recognize variables in a formula.<br><br>2. Try not to finish too many things in one step. (finish $\frac{dh}{dt}$ and the chain rule in one step) |

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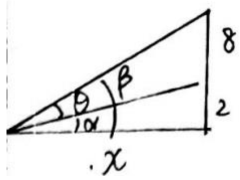
<tr><th colspan="4">MATHEMATICS Extension 1 Question 9</th></tr>
<tr><th>Q</th><th>Suggested Solutions</th><th>Mark</th><th>Comments</th></tr>
<tr>
<td>C i)</td>
<td>

$\tan \alpha = \frac{2}{x}$ $\alpha = \tan^{-1}\left(\frac{2}{x}\right)$

$\tan \beta = \frac{10}{x}$ $\beta = \tan^{-1}\left(\frac{10}{x}\right)$

$\theta = \beta - \alpha = \tan^{-1}\left(\frac{10}{x}\right) - \tan^{-1}\left(\frac{2}{x}\right)$

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<td>1</td>
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**Common mistakes:**

No enough working out. Go straight to the inverse trig expressions before showing the trig ratios. (Proof questions requires you to show the steps that lead to the final statement)

**Suggestion:**

1. Don’t skip necessary steps, especially in a proof question

2. Please refer to which angle by using the name of the angle or label the angles in a diagram, rather than calling them the big angle and the small angle

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<tr>
<td>C ii)</td>
<td>

$\tan \theta = \tan(\beta - \alpha)$

$= \dfrac{\tan\beta - \tan\alpha}{1 + \tan\beta \tan\alpha}$

$= \dfrac{\frac{10}{x} - \frac{2}{x}}{1 + \frac{10}{x} \cdot \frac{2}{x}}$

$= \dfrac{8/x}{1 + \frac{20}{x^2}}$

$= \dfrac{8x}{x^2 + 20}.$

**First mark:** expanding the formula $tan\ (\beta - \alpha)$<br>
**Second mark:** simplify the expression in terms of x

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<td>2<br><br>1<br><br><br>1</td>
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**Common mistakes:**

2. If $\theta = \beta - \alpha$

Then

$tan\ \theta = tan\ \beta - tan\ \alpha$

Or

$tan^{-1}\ \theta = tan^{-1}\ \beta - tan^{-1}\ \alpha$

2. Some students uses Pythagorean theorem but it doesn’t lead to any significant progress

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# MATHEMATICS Extension 1 Question 9

| Q | Suggested Solutions | | Mark | Comments |
|---|---|---|---|---|
| C iii | **Method One:**<br><br>two curves intersect at one point:<br>$\frac{8x}{x^2+20} = k$<br>$kx^2+20k = 8x$<br>$kx^2-8x+20k=0$<br><br>$\Delta = 64-80k^2 \geq 0$<br>$k^2 \leq \frac{4}{5}$<br>$-\frac{2}{\sqrt{5}} \leq k \leq \frac{2}{\sqrt{5}}$ | **Method two:**<br>$y = \frac{8x}{x^2+20}$<br>$y' = \frac{8(x^2+20)-2x(8x)}{(x^2+20)^2}$<br>$= \frac{8x^2+160-16x^2}{(x^2+20)^2}$<br>$= \frac{8(20-x^2)}{(x^2+20)^2}$<br><br>When the curve intersects $y=k$ at one point:<br>it is either maximum or minimum<br>$y'=0 \Rightarrow 20-x^2=0 \Rightarrow x = \pm 2\sqrt{5}$<br>$x=2\sqrt{5},\ y=\frac{2}{\sqrt{5}}$; $x=-2\sqrt{5},\ y=\frac{-2}{\sqrt{5}}$<br>$\therefore \frac{-2}{\sqrt{5}} \leq y \leq \frac{2}{\sqrt{5}}$<br>To make sure two curves have minimum one inters<br>$\frac{-2}{\sqrt{5}} \leq k \leq \frac{2}{\sqrt{5}}$ | **2**<br><br>**1**<br><br><br><br><br>**1** | **Common mistake:**<br><br>Do not show working out for finding the value of $y = \pm\frac{2}{\sqrt{5}}$ but take it as if it is given |
| C iv | **Method One:**<br>$\frac{8x}{x^2+20} = \frac{2}{\sqrt{5}}$<br>$\frac{4x}{x^2+20} = \frac{1}{\sqrt{5}}$<br>$4\sqrt{5}x = x^2+20$<br>$x^2-4\sqrt{5}x+20=0$<br>$x = \pm 2\sqrt{5}$ (rem<br>**(remove the negative answer because** $x \geq 0$**)**<br><br>**When** $x = 2\sqrt{5}$**, y has the maximum value, according to the graph**<br><br>**When** $x \geq 0, y \geq 0$, $\boldsymbol{\theta} = \tan^{-1}y$ **is a monotonic increasing function.**<br><br>**That is, when y has the maximum value,** $\boldsymbol{\theta}$ **has the maximum value.**<br><br>**Therefore, when** $x = 2\sqrt{5}$, $\boldsymbol{\theta}$ **has the maximum value.** | **Method two:**<br><br>**When** $x = 2\sqrt{5}$**, y has the maximum value (calculation of x is shown c iv)**<br><br>**When** $x \geq 0, y \geq 0$, $\boldsymbol{\theta} = \tan^{-1}y$ **is a monotonic increasing function.**<br><br>**That is, when y has the maximum value,** $\boldsymbol{\theta}$ **has the maximum value.**<br><br>**Therefore, when** $x = 2\sqrt{5}$, $\boldsymbol{\theta}$ **has the maximum value.** | **2**<br><br>**1**<br><br><br><br>**1** | **Common mistakes:**<br><br>1. Do not show working out for finding the value of $x = 2\sqrt{5}$ but take it as if it is given<br><br>2. Majority of students do not recognise that it is a composite function. The nature of the external function $\boldsymbol{\theta} = \tan^{-1}y$ needs to be considered as well. (That is, $\theta = tan^{-1}y$ is a monotonic increasing function)<br><br>3. If the correct working out of $x = 2\sqrt{5}$ is included in ciii), students still need to mention that y is max when $x = 2\sqrt{5}$ according to the provided diagram (or an equivalent statement) |

| MATHEMATICS Extension 1 Question 9 | | | |
|---|---|---|---|
| **Q** | **Suggested Solutions** | **Mark** | **Comments** |
| **d i)** | i) $\frac{dP}{dt} = -kBe^{-kt}$.<br>Given $P = A + Be^{-kt}$. $\Rightarrow Be^{-kt} = P - A$.<br>$\therefore \frac{dP}{dt} = -k(P-A)$ satisfy the equation. | 1 | **Suggestion:**<br>Many students treat this proof question as a normal equation proof, thus taking the approach of RHS=..., then sub in the expressions of m and n, which equals LHS.<br><br>**Common mistakes:**<br>Do not have $Be^{-kt} = P - A$ or an equivalent statement. |

| MATHEMATICS Extension 1 Question 9 | | | |
|---|---|---|---|
| **Q** | **Suggested Solutions** | **Mark** | **Comments** |
| **d ii)** | when $t=0$, $P = A + B = n$ ①<br>when $t=1$, $P = A + Be^{-k} = m$. ②<br>from ② $e^{-k} = \frac{m-A}{B}$ ③<br>from ① $B = n - A$ ④<br>③ & ④ $\Rightarrow$ $e^{-k} = \frac{m-A}{n-A}$ ⑤<br>Sub ⑤ into $P = A + Be^{-kt}$<br>$P = A + B\left(\frac{m-A}{n-A}\right)^t$ | **3**<br><br>**1**<br><br>**1**<br><br>**1** | **Common mistakes:**<br><br>**Skip too many steps**<br>$A + B = n$<br>$A + Be^{--kt} = m$<br>Therefore<br>$e^{-kt} = (\frac{Be^{-k}}{B})^t$<br>$= (\frac{m-A}{n-A})^t$<br><br>**Please show necessary steps. At least, you need to show**<br>$B = n - A$<br>$Be^{-kt} = m - A$<br>**Then**<br>$e^{-kt} = (\frac{Be^{-k}}{B})^t = (\frac{m-A}{n-A})^t$<br><br>**Suggestion:**<br><br>Many students treat this proof question as a normal equation proof, thus taking the approach of RHS=..., then sub in the expressions of m and n, which equals LHS.<br><br>Please be aware that this question requires you to prove a formula, therefore starting from original expression $P = A + Be^{-kt}$ is a more mathematically correct approach.<br><br>**Long method:**<br>Find the expression of k in terms of m and n, and then sub it into the original expression $P = A + Be^{-kt}$ |

| | | | |
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| | | | **Short method:**<br>Compare the original $P$formula and the new $P$formula, the only difference is $e^{-kt}$ and $\frac{m-A}{n-A}$<br>Therefore, proving $e^{-kt} = \frac{m-A}{n-A}$ should be the focus of the entire problem-solving process |

## MATHEMATICS Extension 1 Question 9

| Q | Suggested Solutions | Mark | Comments |
|---|---|---|---|
| | **Method One**<br>$A = 100$<br>$P = 100 + Be^{-kt}$<br>$t = 0,\ P = 650 = 100 + B \Rightarrow B = 550$<br>$n = 650,\ m = 500$ | 2 | **Common mistakes:**<br>1. Do not realise this is an inequality question, and the final value needs to be greater than 7.5298. Take 7.52 or 7.5 as the final answer |
| | $P = 100 + 550\left(\frac{500-100}{650-100}\right)^t$<br>$= 100 + 550\left(\frac{8}{11}\right)^t$<br>$P < 150$<br>$100 + 550\left(\frac{8}{11}\right)^t < 150$<br>$\left(\frac{8}{11}\right)^t < \frac{1}{11}$<br>$\ln\left[\left(\frac{8}{11}\right)^t\right] < \ln\frac{1}{11}$<br>$t \times \ln\frac{8}{11} < \ln\frac{1}{11}$<br>$t > \dfrac{\ln\frac{1}{11}}{\ln\frac{8}{11}} \quad \left(\frac{8}{11} < 1,\ \ln\frac{8}{11} < 0\right)$ | 1 | 2. Forget to change the inequality sign when both sides divide by a negative number.<br>3. Myth! Write 500 instead of 550 in the formula<br>4. Calculation mistakes |
| | either $t \geq 7.53$ (2dp) $\quad (7.5298\ldots)$ | 1 | **Suggestion:**<br>This is an inequality question. If equations are used instead of inequalities, please find out whether the value should be greater or smaller than the solution |
| | **Method two:**<br>$A = 100$<br>$P = 100 + Be^{-kt}$<br>$t = 0,\ P = 650 = 100 + B \Rightarrow B = 550$<br>$n = 650,\ m = 500$ | | **Note:** |
| | $500 = 100 + 550e^{-k}$<br>$e^{-k} = \frac{400}{550} = \frac{8}{11} \Rightarrow k = -\ln\left(\frac{8}{11}\right)$<br>$150 > 100 + 550e^{-kt}$<br>$e^{-kt} < \frac{50}{550} = \frac{1}{11}$<br>$-kt < \ln\left(\frac{1}{11}\right)$ | 1 | Some students use 149 instead of 150 for the second part of the calculation, which is fine, because the number of koalas needs to be a positive integer and below 150 mean equal to or smaller than 149. |
| | $t > \ln\left(\frac{1}{11}\right) \div (-k) \quad (-k < 0)$<br>$\geq \ln\left(\frac{1}{11}\right) \div \left(+\ln\frac{8}{11}\right)$ | 1 | |
| | **Final answer: 7.53 or 8** | | |

## General suggestions for working out:

1. Make sure your handwriting is easy to read
2. Don't change the lower-case letters to capital letters, vice versa
3. Only one column per page
4. If two or more columns are used, please draw a dividing line and arrows to show the question order.
5. Label the questions number and page number
6. Sort your answer sheets in order before your staple them together