
Student Number

2024 GLENWOOD HIGH SCHOOL

Year 11-Yearly Examination

Assessment Task 3

Mathematics Extension 1

General Instructions

* Reading Time – 5 minutes
* Working time – 1.5 hours
* Write using black pen
* NESA approved calculators may be used
* A reference sheet is provided
* For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 50

Section I – 6 marks (pages 3 – 5)

* Attempt Questions 1-6
* Allow about 10 minutes for this section

Section II – 44 marks (pages 6 – 11)

* Attempt Questions 7 – 9
* Allow about 1 hours and 20 minutes for this section

Section I
6 marks
Attempt Questions 1 – 6.
Allow about 10 minutes for this section.
Use the multiple-choice answer sheet for Questions 1 – 6.

1. Given the letters ABCDEFG, how many arrangements of the letters in a line are there such that it doesn't start with A or B?

A. 5040

B. 3600

C. 4320

D. 3000

2. For the equation $2x^3 - 4x^2 + 6x - 1 = 0$, with roots α, β and γ, what is the value of

$$\alpha^2 + \beta^2 + \gamma^2 \ ?$$

A. -6

B. -4

C. -2

D. 2

3. What is the sixth term in the expansion of $(2x + 3)^8$?

A. ${}^8C_4\ (2x)^4(3)^4$

B. ${}^8C_4\ (2x)^3(3)^5$

C. ${}^8C_5\ (2x)^2(3)^7$

D. ${}^8C_5\ (2x)^3(3)^5$

4. A circular disc expands as it is heated. The area in cm^2 of the disc increases according to the formula $A = (t^2 + 3t)^2$. What is the rate of increase of the area at $t = 2$ minutes?

A. 20 cm^2 / min

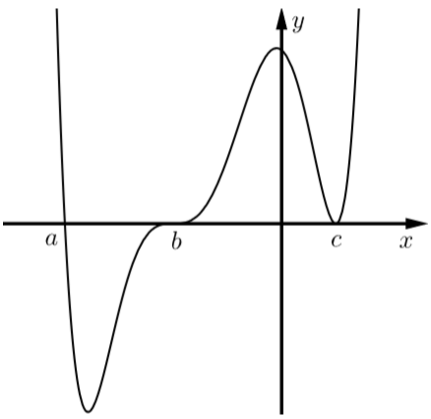
B. 14 cm^2 / min

C. 140 cm^2 / min

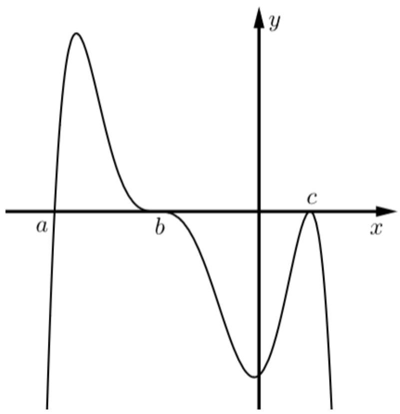
D. 280 cm^2 / min

5. A polynomial is defined as $P(x) = (x-a)(x-b)^3(x-c)^2$. If $ab > 0$ and $bc < 0$ which of the following could be the graph of P(x)?

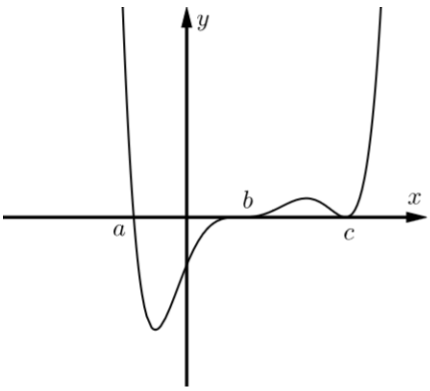
A.



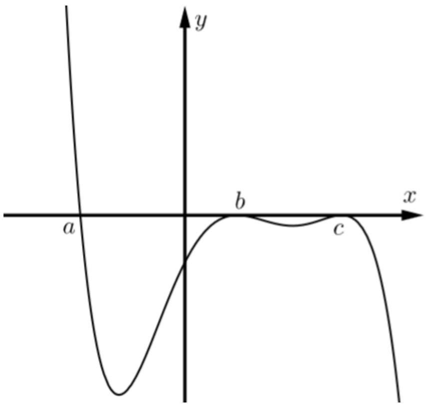
B.



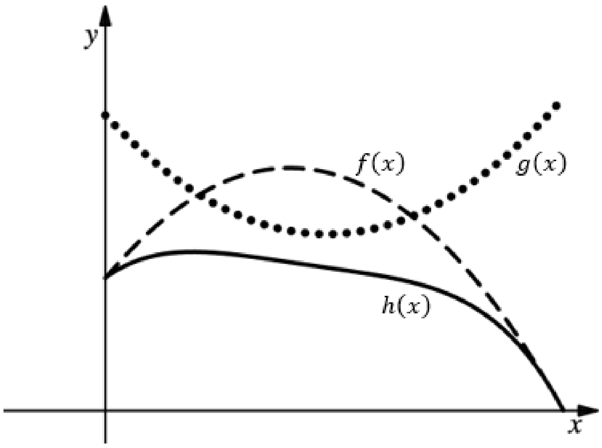
C.



D.



6. The graph shows three functions, $f(x)$, $g(x)$ and $h(x)$.



Which of the following equations can be true for the graphs of $f(x)$, $g(x)$ and $h(x)$, where constant $k > 0$?

A. $h(x) = kf(x)g(x)$

B. $f(x) = kg(x)h(x)$

C. $g(x) = f(x) + kh(x)$

D. $f(x) = g(x) + kh(x)$

End of Section I

Section II
44 marks
Attempt Questions 7 – 9

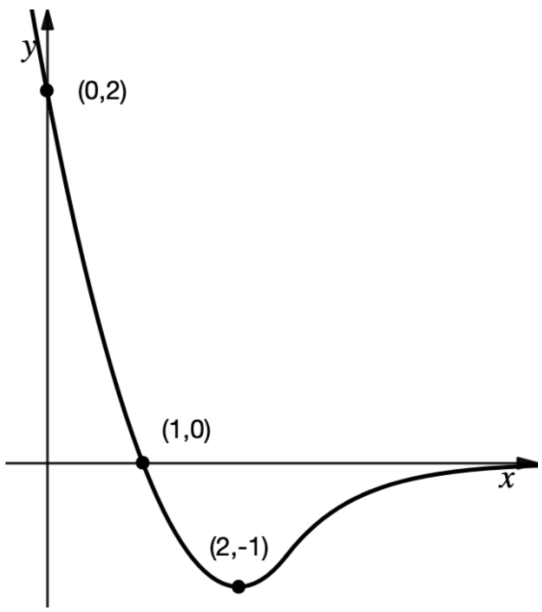
Allow about 1 hour and 20 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.
In Questions 7 – 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 7 (12 marks) Use a new writing booklet.

a) Solve: $|12 - 5x| \leq 3$. **2**

b) The graph shows a function, $f(x)$, with its x-intercept, y-intercept and stationary point labelled.



In your writing booklet, sketch the graph of $y = f(|x|)$ labelling all key features. **2**

Question 7 continues on the next page

Question 7 (continued)

c) Show that: **2**

$$\cos\left(x + \frac{\pi}{3}\right) = \frac{\cos x - \sqrt{3}\sin x}{2}$$

d) Solve: **3**

$$\frac{x(x-3)}{x-1} > -2$$

e) A polynomial $P(x)$ has a remainder of 3 when divided by $(x - 1)$ and $P(-1) = 2$. Find the remainder when the polynomial is divided by $x^2 - 1$. **3**

End of Question 7

Please turn over

Question 8 (16 marks) Use a new writing booklet.

a) There are four boys and four girls sitting around a circular table, including Adam and Anne. How many ways can they be seated if Adam and Anne must be seated next to each other? **1**

b) The parametric equations of a particle are $x = \sin t$ and $y = \cos 2t$, where t is the time in seconds.

i. Find the Cartesian equation. **1**

ii. Give the domain and range of the Cartesian equation. **1**

c) **i.** Give the domain and range of $y = 2\cos^{-1}(x-1)$ **2**

ii. Draw a neat sketch of $y = 2\cos^{-1}(x-1)$ **2**

d) If $x = \tan\theta + \sec\theta$, use the t-formula to show that: **3**

$$\frac{x^2-1}{x^2+1} = \sin\theta$$

Question 8 continues on the next page

Question 8 (continued)

e) Solve the following where n is any positive integer: **2**

$$16(n-1)! = 5n! + (n+1)!$$

f) A particle begins at the origin and moves along the *x*-axis, such that its displacement from the origin (in metres) at a time *t* seconds is given by:

$$x = t^3 - 7t^2 + 12t$$

i. What is the velocity of the particle when it first returns to the origin? **2**

ii. Is the particle speeding up or slowing down as it first returns to the origin? Support your answer with a reason. **2**

End of Question 8

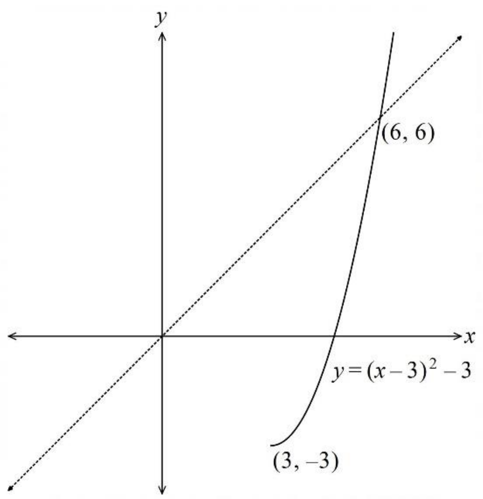
Please turn over

Question 9 (16 marks) Use a new writing booklet.

a) Find the number of ways that a committee of 5 people can be chosen from 6 students and 4 teachers so that the committee includes a majority of students and at least one teacher. **2**

b) **i.** For the function $f(x) = (x-3)^2 - 3;\ \ x \geq 3$, find an equation for $f^{-1}(x)$. **2**

ii. A sketch of $y = f(x)$ is shown below:



Copy the diagram and draw the curve $y = f^{-1}(x)$. **2**

c) Use the pigeonhole principle to determine how many people there would need to be in a crowd to guarantee that there were two people in the crowd with the same initials based on first and last names. For example, Mike Story and Maryanne Smith or Jo Kidd and Jacob Kent. **2**

Question 9 continues on the next page

d) Find the term independent of x in: **2**

$$\left(x + \frac{3}{x^2}\right)^6.$$

e) If α, β, γ are roots of $3x^3 + 8x^2 - 1 = 0$, find the value of $\left(\beta + \frac{1}{\gamma}\right)\left(\gamma + \frac{1}{\alpha}\right)\left(\alpha + \frac{1}{\beta}\right)$. **3**

(Do not find the values of $\alpha, \beta\ and\ \gamma$)

f) Solve the equation $\sin^{-1} x - \cos^{-1} x = \sin^{-1}(5x - 4)$ **3**

END OF PAPER

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1) $5 \times 6! = 3600$ B

2) $2x^3 - 4x^2 + 6x - 1 = 0$

$\alpha + \beta + \gamma = 2$

$\alpha\beta + \alpha\gamma + \beta\gamma = 3$

$\alpha\beta\gamma = \frac{1}{2}$

$(\alpha+\beta+\gamma)^2 = (\alpha+\beta+\gamma)(\alpha+\beta+\gamma)$

$= \alpha^2 + \alpha\beta + \alpha\gamma + \beta\alpha + \beta^2 + \beta\gamma + \alpha\gamma + \beta\gamma + \gamma^2$

$= \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$

$= 2^2 - 2 \times 3 = 4 - 6 = -2$ C

3) $T_6 = \binom{8}{5}(2x)^{8-5}(3)^5$ D

4) $\frac{dA}{dt} = 2(t^2 + 3t)(2t+3)$

at $t = 2$ $\frac{dA}{dt} = 2(2^2 + 3(2))(2(2)+3)$

$= 2(10)(7) = 140 \text{ cm}^2/\text{min}$ C

5) A

6) A

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Question 7:

a) $|12 - 5x| \leq 3$

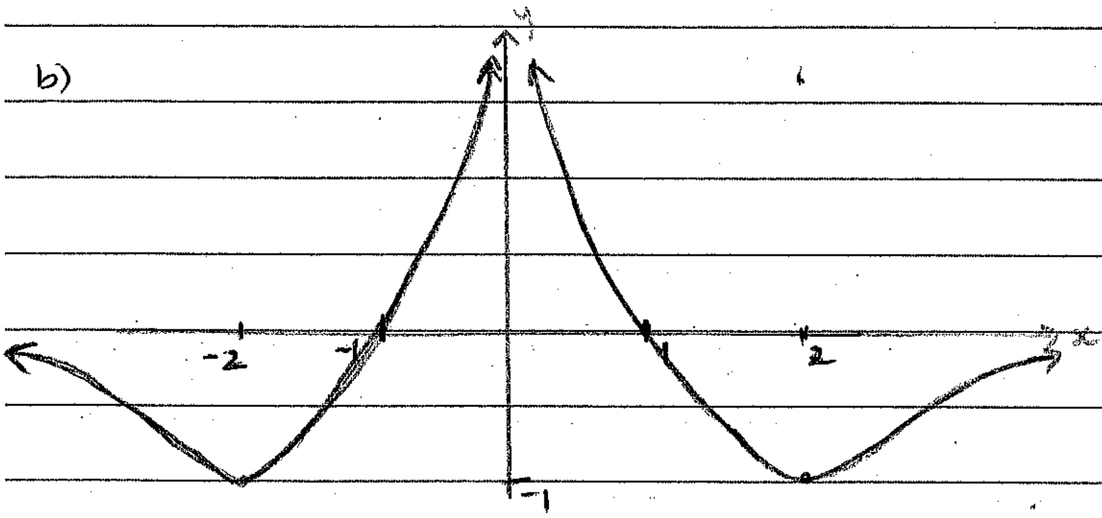
$12 - 5x \leq 3$ $\quad$ $12 - 5x \geq -3$

$-5x \leq -9$ $\quad$ $-5x \geq -15$

$x \geq \frac{9}{5}$ $\quad$ $x \leq 3$

$\frac{9}{5} \leq x \leq 3$

b)



c) $\cos(x + \frac{\pi}{3}) = \cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3}$

$= \cos x (\frac{1}{2}) - \sin x (\frac{\sqrt{3}}{2})$

$= \frac{\cos x}{2} - \frac{\sqrt{3}\sin x}{2}$

$= \frac{\cos x - \sqrt{3}\sin x}{2}$

d) $\frac{x(x-3)}{x-1} > -2$ $\quad$ $x \neq 1$

$x(x-3)(x-1) > -2(x-1)^2$

$x(x-3)(x-1) + 2(x-1)^2 > 0$

$(x-1)(x(x-3) + 2(x-1)) > 0$

$(x-1)(x^2 - 3x + 2x - 2) > 0$

$(x-1)(x^2 - x - 2) > 0$

$(x-1)(x-2)(x+1) > 0$

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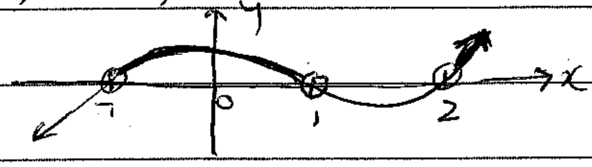
Q7 d contd

$(x-1)(x-2)(x+1) > 0$



$\therefore$ solution is $\underline{-1 < x < 1 \; ; \; x > 2}$

e) $P(1) = 3$ $\quad P(-1) = 2$ $\quad R = ?$

$P(x) = (x+1)(x-1)Q(x) + R$

(this can be $ax+b$ - a linear as dividing by a quadratic)

$P(1) = 3 = 0 + a + b$ ①

$P(-1) = 2 = -a + b$ ②

$5 = 2b$

$\frac{5}{2} = b$

from ② $\quad 2 = -a + b$

$2 = \frac{5}{2} - a$

$\frac{1}{2} = a$

$\therefore$ remainder is $ax+b$ so $\frac{x}{2} + \frac{5}{2}$

Question 8:

a) $6! \times 2! = 1440$

b) $x = \sin t$ $\quad y = \cos 2t$

(i) $y = 1 - 2\sin^2 t$

$= 1 - 2x^2$

(ii) domain $-1 \le x \le 1$ or $[-1, 1]$ $\quad$ range $-1 \le y \le 1$ or $[-1, 1]$

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Q8 continued

c) i) $y = 2\cos^{-1}(x-1)$

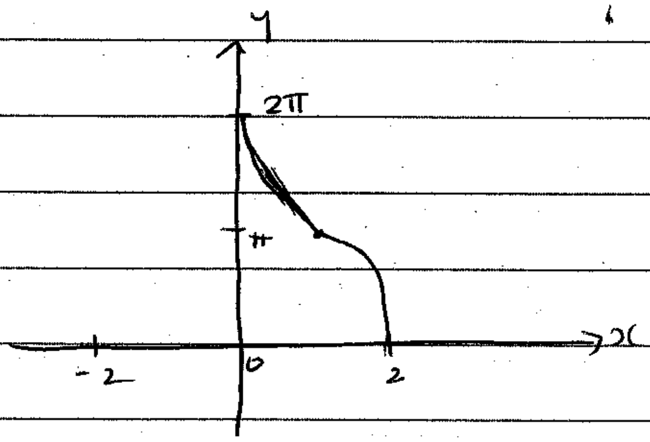
domain: $-1 \leq x-1 \leq 1$

$0 \leq x \leq 2$

range: $0 \leq \cos^{-1}(x-1) \leq \pi$

$0 \leq 2\cos^{-1}(x-1) \leq 2\pi$

(ii)



d) $x = \tan\theta + \sec\theta$

Show $\frac{x^2-1}{x^2+1} = \sin\theta$

$\cos\theta = \frac{1-t^2}{1+t^2}$ $\sec\theta = \frac{1+t^2}{1-t^2}$ $\tan\theta = \frac{2t}{1-t^2}$ $\sin\theta = \frac{2t}{1+t^2}$

$x = \tan\theta + \sec\theta$

$= \frac{2t}{1-t^2} + \frac{1+t^2}{1-t^2}$

$= \frac{t^2+2t+1}{1-t^2}$

$= \frac{(t+1)^2}{(1-t)(1+t)} = \frac{1+t}{1-t}$

Q8 continued

8d) LHS $= \dfrac{x^2-1}{x^2+1}$

$= \dfrac{\left(\frac{1+t}{1-t}\right)^2 - 1}{\left(\frac{1+t}{1-t}\right)^2 + 1}$

$= \dfrac{(1+t)^2 - (1-t)^2}{(1-t)^2} \times \dfrac{(1-t)^2}{(1+t)^2 + (1-t)^2}$

$= \dfrac{1+2t+t^2 - (1-2t+t^2)}{1+2t+t^2+1-2t+t^2}$

$= \dfrac{4t}{2+2t^2} = \dfrac{2t}{1+t^2} = \sin\theta = \text{RHS}$

8e) $16(n-1)! = 5n! + (n+1)!$

$16(n-1)! = 5n(n-1)! + (n+1)(n)(n-1)!$

$16(n-1)! = (n-1)!(5n + n^2 + n)$

$16(n-1)! - (n-1)!(6n + n^2) = 0$

$(n-1)!(16 - n^2 - 6n) = 0$

$(n-1)! = 0$ not possible

$-n^2 - 6n + 16 = 0$

$n^2 + 6n - 16 = 0$

$(n+8)(n-2) = 0$

$n = -8, 2$ n cannot be negative

$\therefore n = 2$

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8f) $x = t^3 - 7t^2 + 12t$

$= t(t^2 - 7t + 12)$

$= t(t-3)(t-4)$

at origin when $x = 0$

$x = 0$ when $t = 0, 3, 4$

(i) First time it will be back at initial position is at $t = 3$

$\dot{x} = 3t^2 - 14t + 12$

$t = 3 \quad \dot{x} = 3(3)^2 - 14(3) + 12$

$= -3\text{ m/s}$

(ii) $x = 0 \quad \dot{x} = -3\text{ m/s}$

$\dot{x} = 3t^2 - 14t + 12$

$\ddot{x} = 6t - 14$

$t = 3 \quad \ddot{x} = 6(3) - 14$

$= 4\text{ m/s}^2$

$\dot{x}$ and $\ddot{x}$ have opposite directions, so particle is slowing down.

Question 9:

a) Need 3 or 4 students and at least 1 teacher.

${}^6C_3 \times {}^4C_2 + {}^6C_4 \times {}^4C_1 = 180$

b (i) $f(x) = (x-3)^2 - 3 \qquad x \geq 3$

$x = (y-3)^2 - 3$

$\pm\sqrt{x+3} = y - 3$

$3 \pm \sqrt{x+3} = y$

$3 + \sqrt{x+3} = y \qquad (y \geq 3)$

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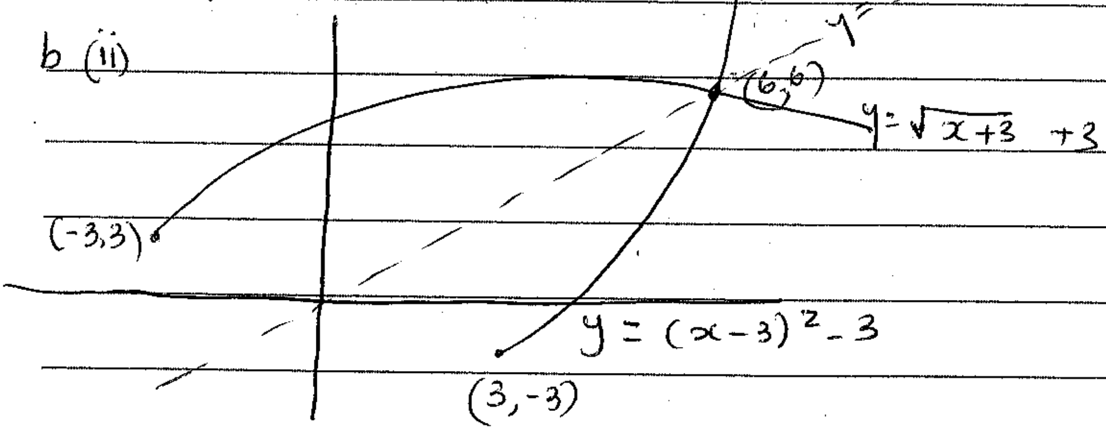
Question 9 continued:

b (ii)



c) 26 possible first and 26 possible second initials.
Number of combinations of these initials is
$26 \times 26 = 676$ (This is the number of different initials possible) - no. of pigeon holes.
So if there were 677 (or more) people in the crowd, it would be guaranteed that 2 people had the same initials.

d) $\left(x + \frac{3}{x^2}\right)^6$

$$T_{r+1} = \binom{6}{r}(x)^{6-r}\left(3x^{-2}\right)^r$$
$$= \binom{6}{r} 3^r x^{6-r-2r}$$
$$= \binom{6}{r} 3^r x^{6-3r}$$
$$x^{6-3r} = x^0$$
$$6 - 3r = 0$$
$$r = 2$$

independent term will be $\binom{6}{2} 3^2 = 135$

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Question 9:

e) $3x^3 + 8x^2 - 1 = 0$ $\quad a=3 \quad b=8 \quad c=0 \quad d=-1$

Find the value of $\left(\beta+\frac{1}{\gamma}\right)\left(\gamma+\frac{1}{\alpha}\right)\left(\alpha+\frac{1}{\beta}\right)$

$\alpha+\beta+\gamma = \frac{-8}{3}$

$\alpha\beta+\alpha\gamma+\beta\gamma = 0$

$\alpha\beta\gamma = \frac{1}{3}$

$\left(\beta+\frac{1}{\gamma}\right) + \left(\gamma+\frac{1}{\alpha}\right)\left(\alpha+\frac{1}{\beta}\right)$

$= \left(\beta\gamma + \frac{\beta}{\alpha} + \frac{\gamma}{\gamma} + \frac{1}{\gamma\alpha}\right)\left(\alpha+\frac{1}{\beta}\right)$

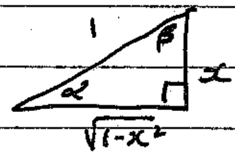
$= \alpha\beta\gamma + \gamma + \beta + \frac{1}{\alpha} + \alpha + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\alpha\beta\gamma}$

$= \alpha\beta\gamma + \alpha+\beta+\alpha + \frac{1}{\alpha\beta\alpha} + \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

$= \frac{1}{3} + \frac{-8}{3} + 3 + \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$ $\qquad (\beta\gamma+\alpha\gamma+\alpha\beta)=0$

$= \frac{2}{3}$

f) $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(5x-4)$



$\sin(\sin^{-1}x - \cos^{-1}x) = \sin(\sin^{-1}(5x-4))$

$\sin\sin^{-1}x \cos\cos^{-1}x - \cos\sin^{-1}x . \sin\cos^{-1}x = 5x-4$

$x^2 - \sqrt{1-x^2} . \sqrt{1-x^2} = 5x-4$

$x^2 - (1-x^2) = 5x-4$

$x^2 - 1 + x^2 = 5x - 4$

$2x^2 - 5x + 3 = 0$

$(2x-3)(x-1) = 0$

$x = 1, \frac{3}{2}$

$\sin^{-1}x$, $\cos^{-1}x$ domain is $-1 \le x \le 1$

$\sin^{-1}(5x-4)$

$-1 \le 5x-4 \le 1$ domain

$\frac{3}{5} \le x \le 1$ ← this domain is common to both

$x=\frac{3}{2}$ is not in this domain so solution is $x=1$

THE END