Student Number

2022	GLENWOOD HIGH SCHOOL

Year 11-Yearly Examination

Assessment Task 3

Mathematics Extension 1

General Instructions

* Reading Time – 10 minutes
* Working time – 1.5 hours
* Write using black pen
* NESA approved calculators may be used
* A reference sheet is provided
* For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 50

Section I – 6 marks (pages 2 – 3)

* Attempt Questions 1-6
* Allow about 10 minutes for this section

Section II – 44 marks (pages 4 – 7)

* Attempt Questions 7 – 9
* Allow about 1 hours and 20 minutes for this section

Section I

6 marks

Attempt Questions 1 – 6.

Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 6.

1. In how many distinct ways can the letters of the word POLLUTION be arranged?

A. 4 578

B. 6 890

C. 90 720

D. 362 880

2. What is the coefficient of x^4 in the expansion of $(2x - 1)^{10}$?

A. -210

B. 210

C. $1\ 680$

D. $3\ 360$

3. A function, $f(x)$, is said to be a self-inverse function if the inverse exists and

$$f(x) = f^{-1}(x).$$

Which of the following functions is considered a self-inverse function?

A. $y = e^x$

B. $y = x + 2$

C. $y = x^2 + 2$

D. $y = -x + 2$

4. The polynomial $P(x) = x^4 - kx^3 - 2x + 33$ has $(x - 3)$ as a factor. What is the value of k?

A. 4

B. 5

C. -4

D. -5

5. A particle is moving in a straight line so that its displacement x after time t is given by $x = 2t^2 - \frac{1}{t}$

What is the acceleration of the particle after 1 second?

A. 1 m/s^2

B. $1\frac{1}{2}$ m/s^2

C. 2 m/s^2

D. 5 m/s^2

6. Mitchell wins a prize which consists of a bundle of fifty $20 notes. He wants to share the money among himself and his six siblings.

He does this by writing the seven names on cards which he places on a table. He then puts a $20 note on each card, until he reaches the end of the row and then goes back to the start and repeats this process until all the notes are distributed.

What is the most money that any one person will receive?

A. $140.00

B. $160.00

C. $180.00

D. $200.00

End of Section I

Section II

44 marks

Attempt Questions 7 – 9

Allow about 1 hour and 20 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

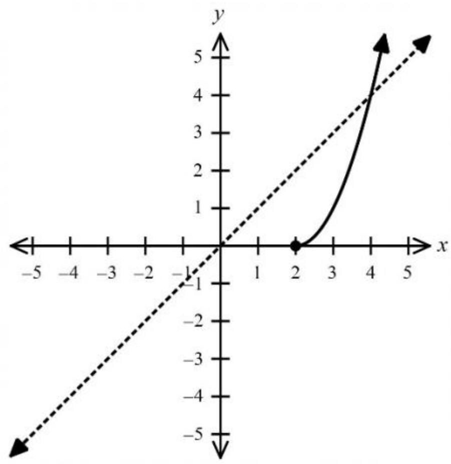
In Questions 7 – 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 7 (13 marks) Use a new writing booklet.

a) Solve the inequality: **3**

$$\frac{x}{5-x} \geq 1$$

b) The graph of $y = f(x) = (x-2)^2;\ [2, \infty)$ is shown below, along with the line $y = x$.

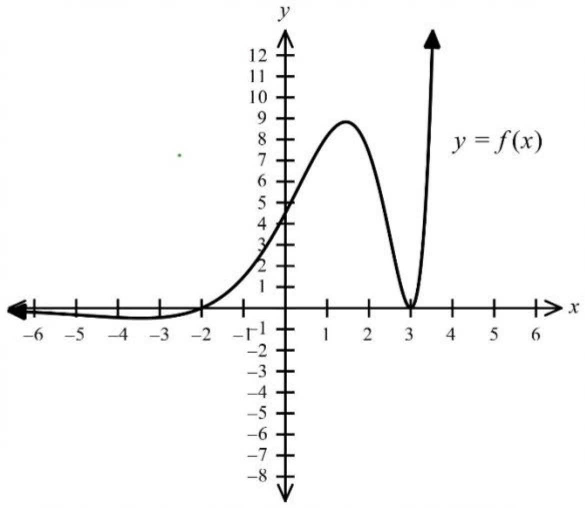


Write the equation of the inverse function in the form $y = f^{-1}(x)$ and state its domain. **2**

c) Find the cartesian equation of the circle with the parametric equations **3**

$$x = 10 + 4cos\theta \text{ and } y = -8 + 4sin\theta$$

d) The graph of $y = f(x)$ is shown below, for the domain (-6, 6).



i) Draw a half-page sketch of $y = |(f(x))|$ showing important details. **1**

ii) Draw a half-page sketch of $y = \dfrac{1}{f(x)}$ showing any asymptotes and intercepts. **2**

e) Show that **2**

$$\cos 20° \sin 10° = \frac{1 - 2\sin 10°}{4}$$

End of Question 7

Question 8 (17 marks) Use a new writing booklet.

a) $P(x)$ has a remainder 2 when divided by $(x-3)$ and a remainder -5 when divided by $(x-2)$. **3**

Find the remainder when $P(x)$ is divided by $x^2 - 5x + 6$.

b) The displacement (in metres) from the origin, after t seconds, of a particle moving along the number line is given by:

$x = t^3 - 10t^2 + 12t + 24\,.$

i) Find the times when the particle is at rest. **2**

ii) Find the velocity of the particle at the time where there is no accelerating force acting on it. **2**

c) i) State the domain and range of the function $f(x) = 3\cos^{-1}(1-2x)$. **2**

ii) Hence, sketch $f(x) = 3\cos^{-1}(1-2x)$. Label any intercepts with the coordinate axes. **2**

d) Eight differently coloured points are to be equally spaced around two identical circles. Each circle is to have four different coloured points. **2**

How many different ways can these coloured points be arranged?

e) Prove that **4**

$$\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$$

Hence, show that

$$\tan\frac{\pi}{12} = 2-\sqrt{3}$$

End of Question 8

Question 9 (14 marks) Use a new writing booklet.

a) Show that $\sec(\tan^{-1} p) = \sqrt{1+p^2}$. **2**

b) If α, β, γ are the roots of the equation $2x^3 - 6x^2 + 10x - 5 = 0$, find the values of:

i) $\alpha + \beta + \gamma$ **1**

ii) $\alpha\beta + \alpha\gamma + \beta\gamma$ **1**

iii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

c) Show that for all integers $n \geq 3$, **3**

$$\binom{n+2}{3} - \binom{n}{3} = n^2$$

d) The rate of change of the temperature T °C of a body after t minutes in an environment where there the surrounding temperature is T_S °C is given by:

$$\frac{dT}{dt} = -k(T - T_S) \text{ where } k \text{ is some constant}$$

Garry takes out a small tub of ice-cream from the deep freezer which was set to -40 °C and finds that after one minute, the ice-cream warms to -30 °C.

Garry knows that the ideal temperature to enjoy his ice-cream is between -15 °C and -5 °C. The temperature in the room is kept at a constant 20 °C.

i) Show that the solution $T = T_S + Ae^{-kt}$, where A is some constant, satisfies the differential equation $\frac{dT}{dt} = -k(T - T_S)$. **1**

ii) Find the exact values of A and k. **2**

iii) Garry begins to eat his ice-cream when it is exactly -15°C cold. How long does Garry have to enjoy his ice-cream before it becomes too warm for his liking. Give your answer correct to the nearest second. **2**

End of Question 9

END OF PAPER

11X1 — Yearly 2022

1). 9 letters, 2 O's, 2 L's

$\frac{9!}{2! \times 2!} = 90720$ (C)

2). $(2x-1)^{10}$

$T_{k+1} = \binom{10}{k}(2x)^{10-k}(-1)^k$

$= \binom{10}{k}(2)^{10-k}(-1)^k x^{10-k}$

$10 - k = 4$

$k = 10 - 4 = 6$ (D)

Coeffient $= \binom{10}{6}(2)^{10-6}(-1)^6$

$= \binom{10}{6}(2)^4 = 3360$

3). $y = -x + 2$

Interchanging x & y, $x = -y + 2$

$y = 2 - x$

$\therefore$ (D)

4) $P(3) = (3)^4 - k(3)^3 - 2(3) + 33$

$= 81 - 27k - 6 + 33$

$= 108 - 27k$

$108 - 27k = 0$

$-27k = -108$

$k = \frac{-108}{-27} = 4$ (A)

5). $x = 2t^2 - \frac{1}{t}$

$v = 4t + \frac{1}{t^2}$

$a = 4 - \frac{2}{t^3}$

At $t=1$, $a = 4 - \frac{2}{1} = 2$ (C)

@ rest $v = 0$

$\frac{1-2t^2}{\sqrt{1-t^2}} = 0$

$1 - 2t^2 = 0$

$-2t^2 = -1$

$t^2 = 1/2$

$t = \pm\frac{1}{\sqrt{2}}$ since $t \geq 0$, $t = \frac{1}{\sqrt{2}}$ (C)

6). $\frac{50}{7} = 7.2$

∴ Atleast one person should get 8 notes

$8 \times 20 = 160$ (B)

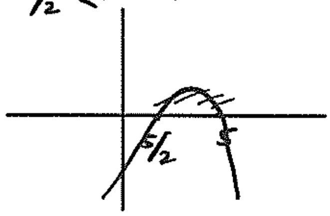
Q7)

a) $\frac{x}{5-x} \geq 1$ $\quad x \neq 5$

$\frac{x(5-x)^2}{5-x} \geq 1(5-x)^2$

$x(5-x) \geq (5-x)^2$

$$x(5-x) - (5-x)^2 \geq 0$$
$$(5-x)(x-(5-x)) \geq 0$$
$$(5-x)(2x-5) \geq 0$$
$$5/2 \leq x < 5$$



b). $f(x) = (x-2)^2 ; \; [2, \infty]$

Interchanging x & y

$$x = (y-2)^2$$
$$y-2 = \pm\sqrt{x}$$
$$y = \pm\sqrt{x} + 2$$

In the restricted domain

$$f^{-1}(x) = \sqrt{x} + 2$$
$$x \in [0, \infty)$$

c) $x = 10 + 4\cos x$

$y = -8 + 4\sin x$

Rearranging

$$\cos x = \frac{x-10}{4} \quad \text{——} (1)$$

$$\sin x = \frac{y+8}{4} \quad \text{——} (2)$$

$(1)^2 + (2)^2$

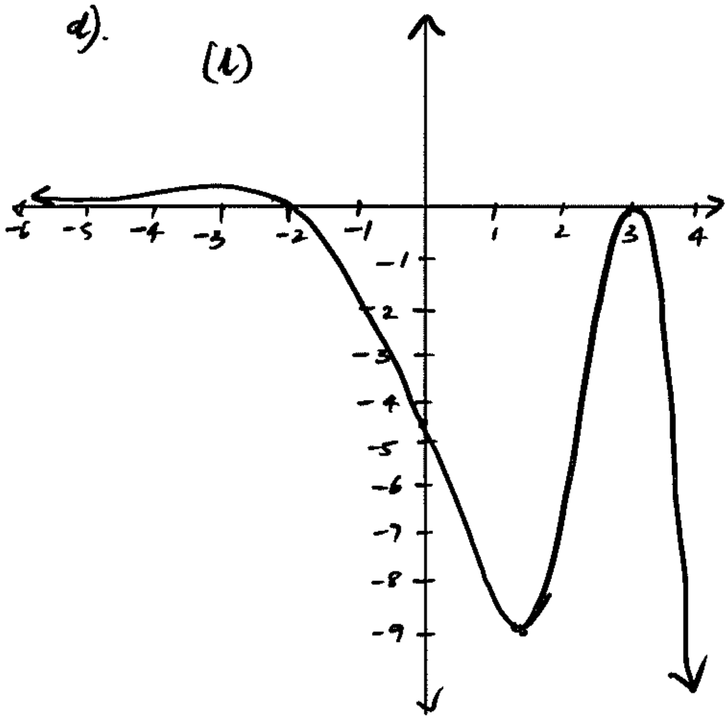
$$\sin^2 x + \cos^2 x = \left(\frac{y+8}{4}\right)^2 + \left(\frac{x-10}{4}\right)^2$$

$$1 = \frac{(y+8)^2 + (x-10)^2}{16}$$

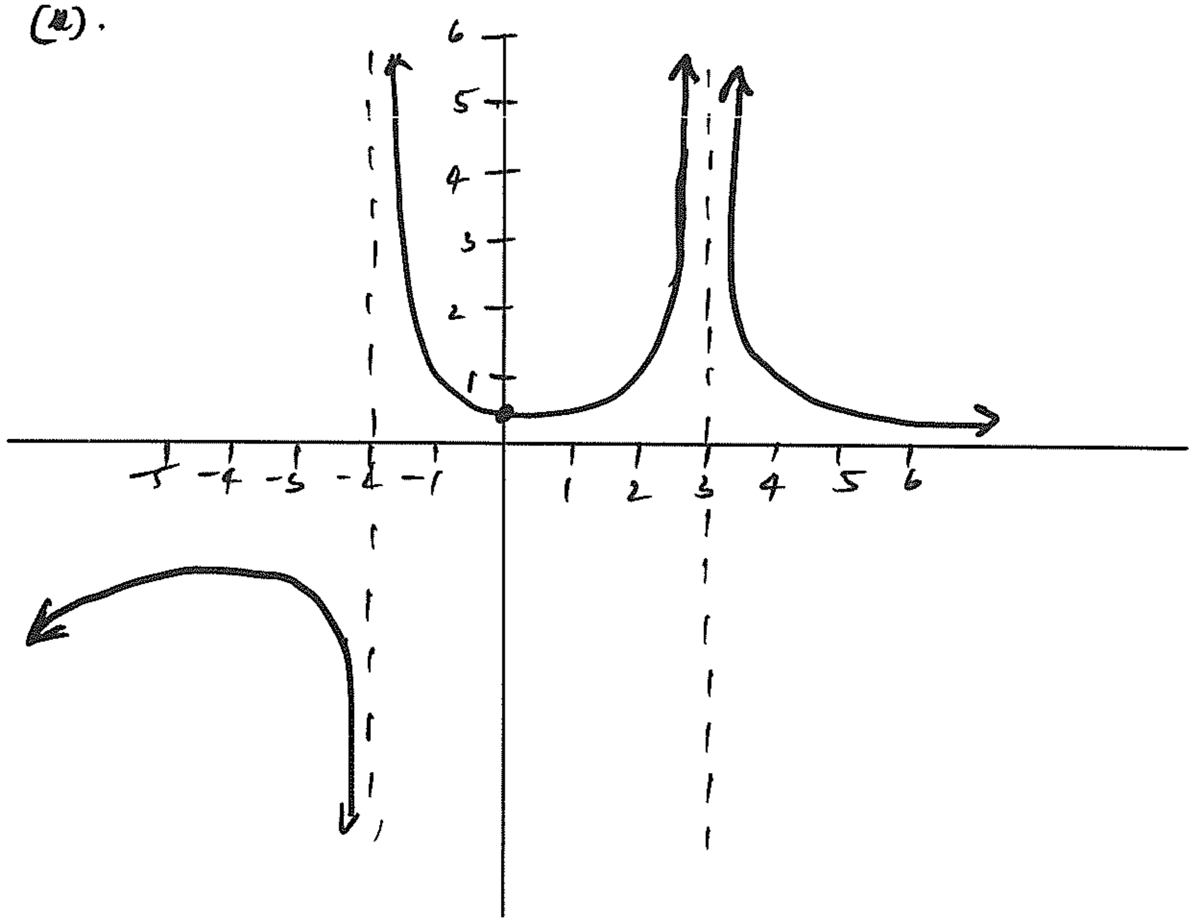
$$\therefore (x-10)^2 + (y+8)^2 = 16$$

d).

(i)



(ii).



e) $\cos 20 \sin 10 = \frac{1}{2}(\sin 30 - \sin 10)$

$$= \frac{1}{2}\left(\frac{1}{2} - \sin 10\right)$$

$$= \frac{1}{2}\left(\frac{1 - 2\sin 10}{2}\right)$$

$$= \frac{1 - 2\sin 10}{4}$$

8). $P(3) = 2$

$P(2) = -5$

$P(x) = (x^2 - 5x + 6)\, Q(x) + ax + b$

$P(x) = (x-3)(x-2)\, Q(x) + ax + b$

$P(3) = a(3) + b = 2$

$3a + b = 2$ ——— (1)

$P(2) = a(2) + b = -5$

$2a + b = -5$ ——— (2)

(1) − (2) $a = 7$

$b = 2 - 3(7)$

$= 2 - 21 = -19$

Remainder $= 7x - 19$

b). $x = t^3 - 10t^2 + 12t + 24$

$v = 3t^2 - 20t + 12$

@ rest, $v = 0$

$3t^2 - 20t + 12 = 0$

$(3t-2)(t-6) = 0$

$t = 2/3,\ 6$

$3t \quad -2$

$t \quad -6$

(ii). $a = 6t - 20$

when $a = 0$

$6t - 20 = 0$

$6t = 20$

$t = 20/6 = 10/3$

when $t = 10/3$

$v = 3\left(\frac{10}{3}\right)^2 - 20\left(\frac{10}{3}\right) + 12$

$= \frac{\cancel{3}(100)}{\cancel{9}3} - \frac{200}{3} + 12$

$= -21\,1/3$

c). (i) $y \in [0, 3\pi]$

$-1 \leq 1 - 2x \leq 1$

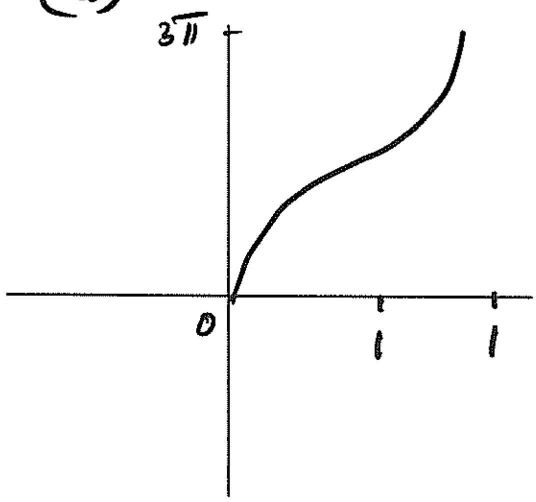
$-2 \leq -2x \leq 0$

$\frac{-2}{-2} \geq x \geq \frac{0}{-2}$

$1 \geq x \geq 0$

$\therefore x \in [0, 1]$

(ii)



d). $\frac{8C_4 \times 3! \times 3!}{2}$

Note:- $8C_4$ → choose 4 points from 8

$3!$ → Arrange 4 points around circle

$3!$ → Arrange the rest 4 around the circle

Divide by 2 as order of circles don't matter c). $$\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$$

<u>LHS</u>

$$\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)}$$

$$= \frac{1-1+2\sin^2 x}{1+2\cos^2 x-1}$$

$$= \frac{\cancel{2}\sin^2 x}{\cancel{2}\cos^2 x}$$

$$= \tan^2 x \quad QED$$

$\tan \frac{\pi}{12}$

Using $\tan^2 x = \dfrac{1-\cos 2x}{1+\cos 2x}$

$$\left(\tan \frac{\pi}{12}\right)^2 = \frac{1-\cos 2\times\pi/12}{1+\cos 2\times\pi/12}$$

$$= \frac{1-\cos \pi/6}{1+\cos \pi/6}$$

$$= \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}$$

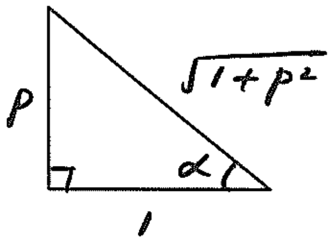
$$= \frac{(2-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$$

$$= \frac{(2-\sqrt{3})^2}{4-3} = (2-\sqrt{3})^2$$

$\tan \pi/12 = 2 - \sqrt{3}$ since $\tan \pi/2 > 0$

Q9) a)

Let $\alpha = \tan^{-1} p$



$$\sec(\tan^{-1} p) = \sec \alpha$$

$$= \frac{1}{\cos \alpha} = \frac{1}{\frac{1}{\sqrt{1+p^2}}} = \sqrt{1+p^2}$$

b) $\alpha+\beta+\gamma = \frac{-b}{a} = \frac{6}{2} = 3$

$\alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a} = \frac{10}{2} = 5$

$$\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$$
$$= (3)^2 - 2(5)$$
$$= 9 - 10$$
$$= -1$$

c) $\binom{n+2}{3} - \binom{n}{3} = n^2$

<u>LHS</u>

$$\binom{n+2}{3} - \binom{n}{3}$$

$$\frac{(n+2)!}{(n+2-3)! \times 3!} - \frac{n!}{(n-3)! \times 3!}$$

$$\frac{(n+2)!}{(n-1)! \times 3!} - \frac{n!}{(n-3)! \times 3!}$$

$$\frac{(n+2)(n+1)\,n\,\cancel{(n-1)!}}{\cancel{(n-1)!} \times 3!} - \frac{n(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!} \times 3!}$$

$$\frac{(n+2)(n+1)n - n(n-1)(n-2)}{3!}$$

$$= \frac{n\left[(n^2+3n+2) - (n^2-3n+2)\right]}{3!}$$

$$= \frac{n(\cancel{n^2} + 3n \cancel{+2} - \cancel{n^2} + 3n \cancel{-2})}{3!}$$

$$= \frac{6n^2}{3!}$$

$$= \frac{6n^2}{6} = n^2$$

d). $\frac{dT}{dt} = -k(T - T_s)$

At $t=0$, $T = -40$

$t = 1$, $T = -30$

$T_s = 20^\circ$

(i) $T = T_s + Ae^{-kt}$ —— (1)

$\frac{dT}{dt} = Ae^{-kt} \times -k$ —— (2)

from (1) $Ae^{-kt} = T - T_s$

Sub in ②

$$\frac{dT}{dt} = (T - T_s) \times -k$$
$$= -k(T - T_s)$$

(ii) $T = T_s + Ae^{-kt}$

$T = 20 + Ae^{-kt}$

sub $t = 0$, $T = -40$

$-40 = 20 + Ae^{0}$

$-40 - 20 = A$

$A = -60$

$\therefore T = 20 - 60e^{-kt}$

sub $t = 1$, $T = -30$

$-30 = 20 - 60(e^{-k})$

$-50 = -60e^{-k}$

$\frac{-50}{-60} = e^{-k}$

$\ln\frac{5}{6} = \ln e^{-k}$

$$\ln \frac{5}{6} = -k \ln e$$

$$k = -\ln 5/6 = \ln 6/5$$

$$\therefore \; T = 20 - 60 e^{(-\ln 6/5)t}$$

At $T = -15$

$$20 - 60 e^{(-\ln 6/5)t} = -15$$

$$-60 e^{(-\ln 6/5)t} = -35$$

$$e^{(-\ln 6/5)t} = \frac{-35}{-60} = \frac{7}{12}$$

$$(-\ln 6/5)\, t = \ln 7/12$$

$$t = \frac{-1}{\ln 6/5} \times \ln 7/12$$

At $T = -5$

$$20 - 60 e^{(-\ln 6/5)t} = -5$$

$$-60 e^{(-\ln 6/5)t} = -25$$

$$e^{(-\ln 6/5)t} = \frac{-25}{-60} = \frac{5}{12}$$

$$(-\ln 6/5)t = \ln 5/12$$

$$t = \frac{-1}{\ln 6/5} \times \ln 5/12$$

Time Garry have $= \left(\frac{-1}{\ln 6/5} \times \ln 5/12\right) - \left(\frac{-1}{\ln 6/5} \times \ln 7/12\right)$

$$= \frac{1}{\ln 6/5}\left(-\ln 5/12 + \ln 7/12\right)$$

$=$ 1 min 51 sec