

Student No.

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2020 YEAR 11
YEARLY EXAMINATION

Mathematics Extension 1

**General
Instructions**

- Reading time – 10 minutes
- Working time – 1.5 hours
- Write using black pen
- NESA Approved calculators may be used
- A reference sheet is provided
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

**Total marks:
51**

Section I – 6 marks (pages 3 – 5)

- Attempt Questions 1 – 6
- Allow about 10 minutes for this section

Section II – 45 marks (pages 7 – 10)

- Attempt Questions 7 – 9
- Allow about 1 hour and 20 minutes for this section

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Section I**6 marks****Attempt Questions 1–6****Allow about 10 minutes for this section**

Use the multiple-choice answer sheet for Questions 1 – 6.

1. In how many ways can five boys and two girls be arranged in a row if the two girls are together?

- (A) 300
(B) 1440
(C) 10
(D) 10080

2. Which of the following is correct

- (A) $\sin^{-1}(-x) = -\sin^{-1}(x)$
(B) $\cos^{-1}(-x) = \pi + \cos^{-1}(x)$
(C) $\sin^{-1}(-x) = \sin^{-1}(x)$
(D) $\tan^{-1}(-x) = \tan^{-1}(x)$

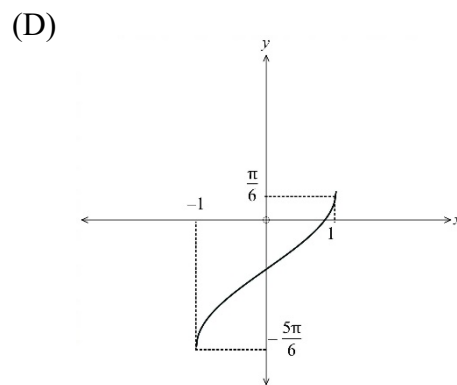
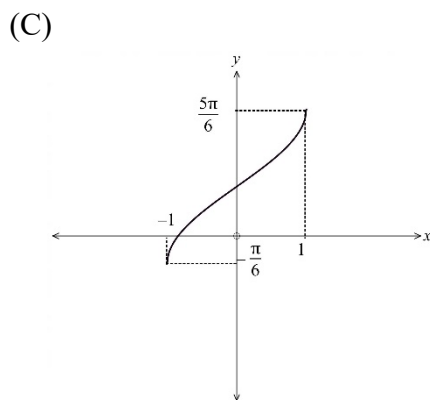
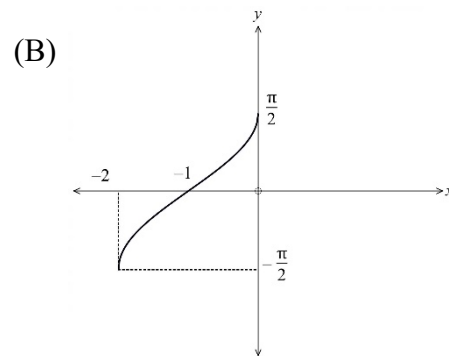
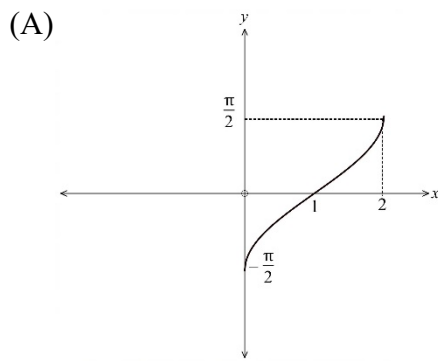
3. What is the exact value of $\cos(15^\circ)$?

- (A) $\frac{\sqrt{6}}{4}$
(B) $\frac{\sqrt{3}}{4}$
(C) $\frac{\sqrt{6} - \sqrt{2}}{4}$
(D) $\frac{\sqrt{6} + \sqrt{2}}{4}$

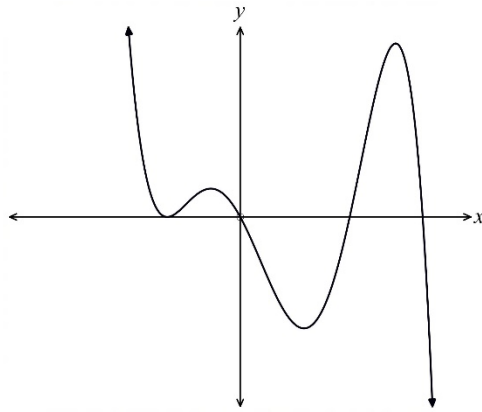
4. It is known that two of the roots of the equation $3x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other. What is the value of k ?

- (A) -2
- (B) 6
- (C) 7
- (D) 17

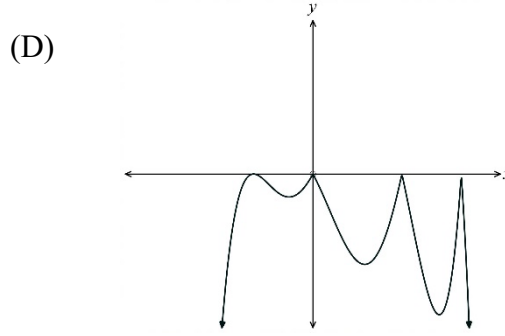
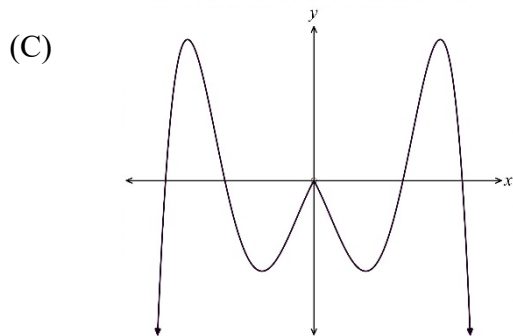
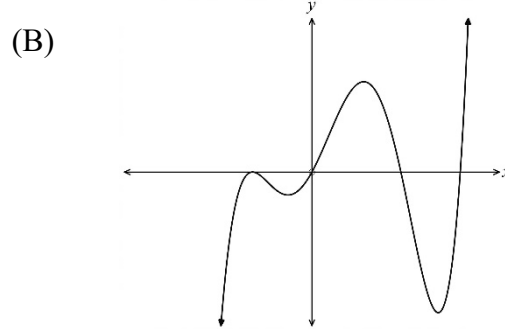
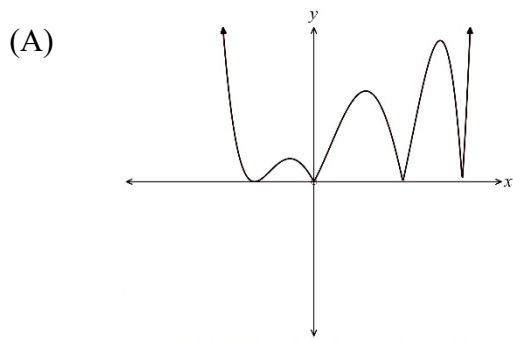
5. Which graph shows the curve $y = \sin^{-1}(x + 1)$?



6. The graph of $y = f(x)$ is shown below.



Which graph shows $y = f(|x|)$?



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Section II**45 marks****Attempt Questions 7 – 9.****Allow about 1 hour and 20 minutes for this section.**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 7 – 9, your responses should include relevant mathematical reasoning and/or calculations.

Question 7 (15 marks) Start a new writing booklet.

- a) There are five women and six men in a group. From this group, a committee of four is to be chosen that contains three women and one man.
- How many different ways can this committee be formed? **1**
 - A particular man and woman must be in the committee. What is the probability that this committee is formed? **1**
- b) Solve the inequality $\frac{4x}{x+3} \geq 1$ **3**
- c) Find the term independent of x in the binomial expansion of $\left(2x - \frac{1}{x^2}\right)^9$ **2**
- d) Let $f(x) = e^{x+2}$.
- Find the inverse function $f^{-1}(x)$. **2**
 - State the domain and range of $f^{-1}(x)$. **1**
- e) Consider the polynomial $P(x) = 2x^3 + 3x^2 - 29x - 60$.
- Use the factor theorem to show that $(x + 3)$ is a factor of $P(x)$ **1**
 - Factorise $P(x)$ completely. **2**
- f) Show that $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$ **2**

End of Question 7

Question 8 (16 marks) Start a new writing booklet.

a) If α, β and γ are the roots of the cubic equation $5x^3 + 2x^2 - x - 3 = 0$, find the value of

i) $\alpha + \beta + \gamma$ 1

ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ 1

iii) $\alpha^2 + \beta^2 + \gamma^2$ 2

iv) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ 1

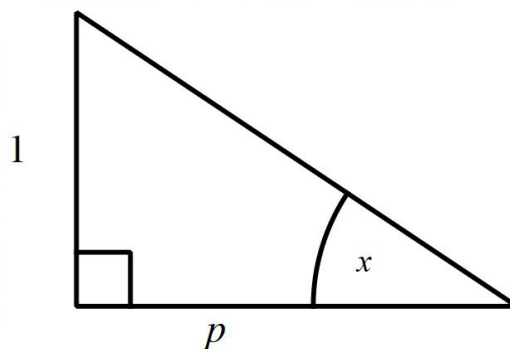
b) Show that $\sin \theta \sin \left(\frac{\pi}{2} - \theta \right) = \frac{1}{2} \cos \left(2\theta - \frac{\pi}{2} \right)$. 2

c)

i) Evaluate $\sin^{-1} \left(\cos \left(\frac{\pi}{6} \right) \right)$ 2

ii) Evaluate $\cos \left(\tan^{-1} \left(\frac{-1}{3} \right) \right)$ in exact value 2

iii) Using the triangle below, or otherwise, simplify $\sin^{-1}(\cos(x))$. 2

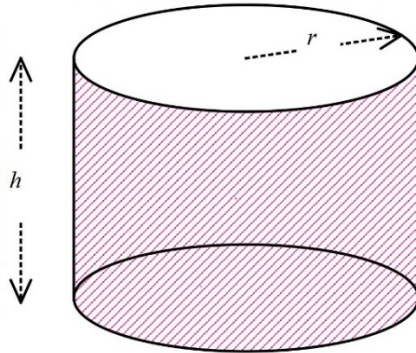


d) A particle is moving in a straight line, so that its displacement x metres from a fixed point on the line at t seconds ($t > 0$) is given by $3t^3 - t^2 - 2t$. Find when the acceleration is positive. 3

End of Question 8

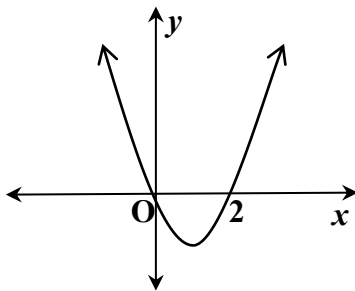
Question 9 (14 marks) Start a new writing booklet.

- a) The solid shown below is a cylinder.



The height of the cylinder is equal to its diameter. In an animation, the model is being scaled up so that its volume is increasing at a steady rate of 120 cm^3 per second.

- i) Write an equation for the volume of the solid in terms of the radius r . 1
 - ii) Find an expression for $\frac{dr}{dt}$ in terms of r . 1
 - iii) Find the rate at which the radius is increasing when it reaches 2 cm. 2
 - iv) What is the radius when $\frac{dr}{dt} = \frac{80}{\pi}$? 1
- b) Given $f(x) = x^2 - 2x$. On separate diagrams sketch the graphs of the following. Indicate all important details.



- i) $y = \sqrt{f(x)}$ 2
- ii) $y = \frac{1}{f(x)}$ 2

Question 9 continues on next page

- c) Newtons law of cooling states that the rate of cooling of a body is given by

$$\frac{dT}{dt} = -k(T - S)$$

where T is the temperature of the body at time t minutes, S is the temperature of the surrounding medium and k is a constant.

Initially, a certain body has a temperature of 60°C and is cooling at a rate of 2°C per minute.

- | | | |
|------|--|----------|
| i) | Verify that $T = S + Ae^{-kt}$ (where A is a constant) is a solution of | 1 |
| | $\frac{dT}{dt} = -k(T - S)$. | |
| ii) | Find the value of k if the temperature of the surrounding medium is 20°C . | 1 |
| iii) | Find the value of A | 1 |
| iv) | How long will it take the temperature of the body to fall to 30°C ?
Answer to the nearest minute. | 2 |

End of Question 9

End of paper

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1) $6P_6 \times 2! = 1440$ (B)

2) $\sin^{-1}(-x) = -\sin^{-1}(x)$ (A)

3) $\cos 15 = \cos(45 - 30)$ (D)

$$= \cos 45 \cos 30 + \sin 45 \sin 30$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{(\sqrt{3}+1)\sqrt{2}}{2\sqrt{2} \times 2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

4) Let the roots be $\alpha, \frac{1}{\alpha}, \beta$

$$3x^3 + x^2 - kx + 6 = 0$$

$$\alpha \cdot \frac{1}{\alpha} \cdot \beta = \frac{-6}{3}$$

$$\beta = -2$$

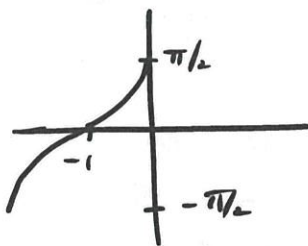
$$3(-2)^3 + (-2)^2 - k(-2) + 6 = 0$$

$$-24 + 4 + 2k + 6 = 0$$

$$2k = 14$$

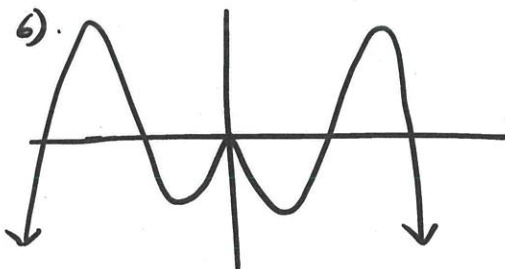
$$k = 7$$

5).



B

6).



C

1 B

2 A

3 D

4 C

5 B

6 C

Question 7

a) i) $5C_3 \times 6C_1 = 60$

ii) $P = \frac{4C_2}{60} = \frac{1}{10}$

b) $\frac{4x}{x+3} \geq 1 \quad x \neq -3$

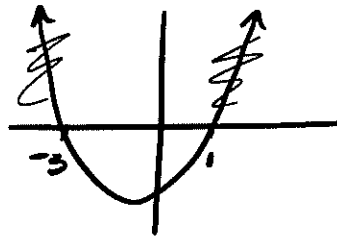
$$4x(x+3) \geq 1(x+3)^2$$

$$4x(x+3) - (x+3)^2 \geq 0$$

$$(x+3)(4x - x - 3) \geq 0$$

$$(x+3)(3x-3) \geq 0$$

$$x < -3 \quad x \geq 1$$



c)
$$\begin{aligned} T_{k+1} &= {}^9C_k (2x)^{9-k} \left(\frac{-1}{x^2}\right)^k \\ &= {}^9C_k (2)^{9-k} (x)^{9-k} (-1)^k (x)^{-2k} \\ &= {}^9C_k (2)^{9-k} (-1)^k (x)^{9-3k} \end{aligned}$$

for independent term $9-3k=0$

$$-3k = -9$$

$$k = 3$$

$$\begin{aligned} \therefore T_4 &= {}^9C_3 (2)^{9-3} (-1)^3 (x)^0 \\ &= -5376 \end{aligned}$$

$$d) (D) f(x) = e^{x+2}$$

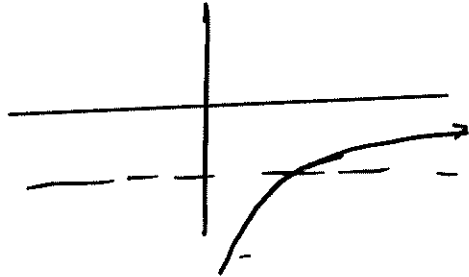
$$x = e^{y+2}$$

$$y+2 = \ln x$$

$$y = \ln x - 2$$

$$x) x \in (0, \infty)$$

$$y \in (-\infty, \infty)$$



$$e) (i) P(x) = 2x^3 + 3x^2 - 29x - 60$$

$$P(-3) = 2(-3)^3 + 3(-3)^2 - 29(-3) - 60$$

$$= 0$$

$\therefore (x+3)$ is a factor

$$(ii) \begin{array}{r} 2x^2 - 3x - 20 \\ x+3 \overline{) 2x^3 + 3x^2 - 29x - 60} \\ \underline{2x^3 + 6x^2} \\ -3x^2 - 29x \\ \underline{-3x^2 - 9x} \\ -20x - 60 \\ \underline{-20x - 60} \\ 0 \end{array}$$

$$\therefore P(x) = (x+3)(2x^2 - 3x - 20)$$

$$= (x+3)(2x+5)(x-4)$$

$$\begin{array}{r} 2x \quad 5 \\ x \quad -4 \end{array}$$

f) ^{show} $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$

LHS

$$\begin{aligned} \cos(2\theta + \theta) &= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta \\ &= (\cos^2 \theta - \sin^2 \theta) \cos \theta - 2 \sin \theta \cos \theta \sin \theta \\ &= \cos^3 \theta - \sin^2 \theta \cos \theta - 2 \sin^2 \theta \cos \theta \\ &= \cos^3 \theta - 3 \sin^2 \theta \cos \theta \\ &= \cos^3 \theta - 3(1 - \cos^2 \theta) \cos \theta \\ &= \cos^3 \theta - 3 \cos \theta + 3 \cos^3 \theta \\ &= 4 \cos^3 \theta - 3 \cos \theta \end{aligned}$$

Question 8

5) a) i) $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-2}{5}$ ✓
 ii) $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = \frac{-1}{5}$ ✓
 iii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$
 $= \left(\frac{-2}{5}\right)^2 - 2\left(\frac{-1}{5}\right)$
 $= \frac{4}{25} + \frac{2}{5} = \frac{14}{25}$ ✓✓

iv) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$
 $\alpha\beta\gamma(\alpha + \beta + \gamma) = \frac{3}{5} \left(\frac{-2}{5}\right) = \frac{-6}{25}$ ✓

2) b) Prove $\sin \theta \sin \left(\frac{\pi}{2} - \theta\right) = \frac{1}{2} \cos \left(2\theta - \frac{\pi}{2}\right)$

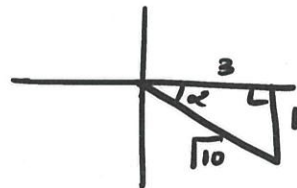
LHS

$$\begin{aligned} \sin \theta \sin \left(\frac{\pi}{2} - \theta\right) &= \sin \theta \cos \theta \\ &= \frac{\sin 2\theta}{2} \\ &= \frac{1}{2} \left[\cos \left(\frac{\pi}{2} - 2\theta\right)\right] \\ &= \frac{1}{2} \left[\cos - \left(2\theta - \frac{\pi}{2}\right)\right] \\ &= \frac{1}{2} \cos \left(2\theta - \frac{\pi}{2}\right) \end{aligned}$$

Need for radian to be kept though out

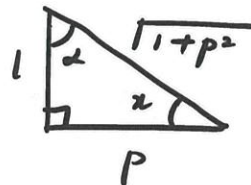
6) c) i). $\sin^{-1} \left(\cos \frac{\pi}{6}\right) = \sin^{-1} \left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

(ii) $\cos \left[\tan^{-1} \left(\frac{-1}{3}\right)\right] = \frac{3}{\sqrt{10}}$



(iii) $\sin^{-1} (\cos x)$

$$\cos x = \frac{P}{\sqrt{1+p^2}}$$



$$\sin^{-1} \left(\frac{P}{\sqrt{1+p^2}}\right) = \alpha = \frac{\pi}{2} - x$$

3) d). $x = 3t^3 - t^2 - 2t$
 $\dot{x} = 9t^2 - 2t - 2$ ✓
 $\ddot{x} = 18t - 2$ ✓
 $18t - 2 > 0$
 $18t > 2$
 $t > \frac{1}{9} \text{ secs}$ ✓

Question 9

a) $V = \pi r^2 h$

$h = 2r$

$V = \pi r^2 \times 2r$

$V = 2\pi r^3$

ii) $V = 2\pi r^3$

$\frac{dV}{dr} = 2\pi \times 3r^2 = 6\pi r^2$

$\frac{dV}{dt} = 120$

$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$\frac{dr}{dt} = \frac{1}{6\pi r^2} \times 120 = \frac{20}{\pi r^2}$

(iii) when $r=2$

$$\frac{dr}{dt} = \frac{20}{\pi(2)^2} = \frac{5}{\pi}$$

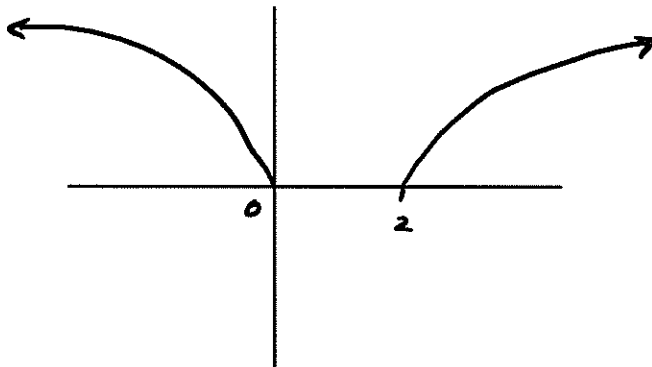
(iv) $\frac{dr}{dt} = \frac{20}{\pi r^2} = \frac{80}{\pi}$

$$\frac{r^2}{20} = \frac{1}{80}$$

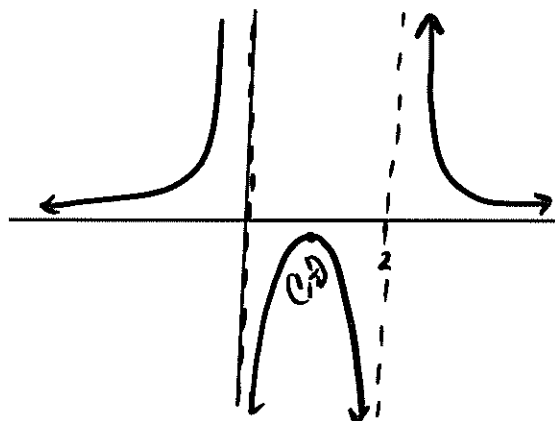
$$r^2 = \frac{20}{80}$$

$$r = \frac{1}{2} \text{ cm.}$$

b). i).



(ii)



(4)

$$c) i) \frac{dT}{dt} = -k(T-S)$$

$$T = S + Ae^{-kt}$$

$$\frac{dT}{dt} = Ae^{-kt} \times -k$$

$$\text{Since } Ae^{-kt} = T - S$$

$$\frac{dT}{dt} = -k(T-S) \quad \checkmark$$

$$(ii) \text{ At } t=0, T = 60^\circ, \frac{dT}{dt} = -2, S = 20$$

$$\frac{dT}{dt} = -k(T-S)$$

$$-2 = -k(60 - 20) \quad \checkmark$$

$$-2 = -k(40)$$

$$k = \frac{2}{40} = \frac{1}{20} = 0.05$$

$$\begin{aligned} \text{(iii)} \quad & \text{At } t=0, \quad S=20, \quad T=60 \\ & 60 = 20 + Ae^0 \\ & 40 = A \end{aligned}$$

$$\text{(iv)} \quad \therefore T = 20 + 40e^{-0.05t}$$

$$\text{At } T=30$$

$$30 = 20 + 40e^{-0.05t}$$

$$10 = 40e^{-0.05t}$$

$$\frac{10}{40} = e^{-0.05t}$$

$$\ln\left(\frac{1}{4}\right) = \ln e^{-0.05t}$$

$$\ln \frac{1}{4} = -0.05t$$

$$t = \frac{-1}{0.05} \ln \frac{1}{4}$$

$$= 28$$