Fort St High School

2021 **Year 11 Assessment Task 3 -** Online examination

Mathematics Extension 1

Total Marks: 51 Working time: 65 minutes

General Instructions

- This is an open book task
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet will be provided
- Show relevant mathematical reasoning and/or calculations

Specific Instructions:

- You will not be able to leave your desk for the duration of the task.
- Mobile phones must be turned off and out of sight.
- Your microphone and cameras must be on, but you can turn the volume on your devices down so that any noise from other students does not disturb you.
- You are not permitted to have headphones/ear buds and if you have long hair, please tie it back.
- You can ask for assistance through the direct chat function of Zoom/Teams or ask as your microphone is on.
- You are to manage your time and make sure you have a timer at hand to keep within the assessment time limit.
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Question 1. (8 marks)

(a) Find the cartesian equation represented by the parametric equations

$x = 2 - p$
$y = p^2 + 1$ **2**

(b) $P(x) = x^3 + 2x^2 - 5x + 1$.

(i) Without using polynomial division, show that $P(x) \div (x-2)$ has remainder 7. **1**

(ii) Verify the result of (i) using polynomial division. **3**

(c) By using a compound angle formula, show that the exact value of $\tan 105^\circ$ is $-\left(2+\sqrt{3}\right)$ **2**

Question 2. (9 marks) *Start a separate Booklet*

(a) Find the domain and range of $y = a\sin^{-1}\left(\frac{x}{b}\right)$, where a and b are constants. **2**

(b) $P(x) = 2x^3 - ax^2 + bx + 3$, where a and b are constants. When $P(x)$ is divided by $x^2 - 1$ the remainder is $-2x - 2$. Find a and b. **3**

(c) (i) Given $f(x) = x^2 - 2x + 2$ for $x \le 1,\ y \ge 1$ what is the domain and range for $f^{-1}(x)$? **1**

(ii) By interchanging x and y, show that inverse function of $f(x) = x^2 - 2x + 2$ for $x \le 1,\ y \ge 1$ is $f^{-1}(x) = 1 - \sqrt{x-1}$ **3**

Question 3. (8 marks) *Start a separate Booklet*

(a) By letting $\alpha = \cos^{-1}\left(\frac{3}{8}\right)$ show, using a right triangle and a double angle formula, that

$\sin\left(2\cos^{-1}\left(\frac{3}{8}\right)\right)$ has exact value $\frac{3\sqrt{55}}{32}$. **3**

(b) (i) Use the t-results to show that $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan\frac{\theta}{2}$ **3**

(ii) Hence, show that $\sqrt{3+2\sqrt{2}} = \tan 67\cdot 5^{\circ}$ **2**

Question 4. (9 marks) *Start a separate Booklet*

(a) If $f\left(x\right)=\ln\left(ex\right)$ and $g\left(x\right)=e^{x-1}$ show that

(i) $f\left(g\left(x\right)\right)=x$ **2**

(ii) $g\left(f\left(x\right)\right)=x$ **2**

(iii) What do the results of (i) and (ii) indicate about $f\left(x\right)$ and $g(x)$? **1**

(b) (i) Show that $\sin\left(\tan^{-1}x\right)=\dfrac{x}{\sqrt{x^2+1}}$ **2**

(ii) Hence or otherwise, show that $f(x)=\sin\left(\tan^{-1}x\right)$ is an odd function. **2**

Question 5. (9 marks) *Start a separate Booklet*

(a) Prove that $\dfrac{x-2}{5x+1} \geq \dfrac{3}{4}$ has solutions $-1 \leq x < -\dfrac{1}{5}$. **3**

(b) Frozen meat is taken out of a freezer by a chef at $-18^{\circ}C$, placed in a room with temperature $25^{\circ}C$ and allowed to defrost. The meat warms according to the law

$$\frac{dT}{dt} = k\left(T_o - T\right)$$

where T is the temperature of the meat, T_o the room temperature, t the time in minutes since meat is taken from the freezer and k is a constant.

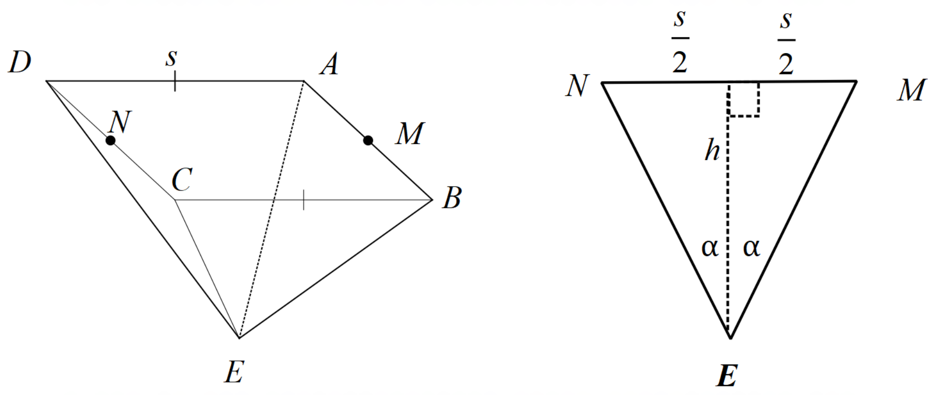
(i) Show that $T = 25 - 43e^{-kt}$ is a solution of the differential equation $\dfrac{dT}{dt} = k\left(T_o - T\right)$. **2**

(ii) After 30 minutes, the chef measures the temperature of the meat to be $-5^{\circ}C$. Show that the exact value of k is $\dfrac{-1}{30}\ln\left(\dfrac{30}{43}\right)$. **2**

(iii) Find the time t that allows the chef to begin cooking the meat once it reaches a temperature of $10^{\circ}C$. Give your answer to the nearest minute. **2**

Question 6. (8 marks) *Start a separate Booklet*

(a) An inverted right square pyramid ABCDE with square sidelength s and midpoints M and N on sides AB and CD, is shown below left. The cross section of the pyramid through N, M and E with height h and semi-apex angle α is shown below right.

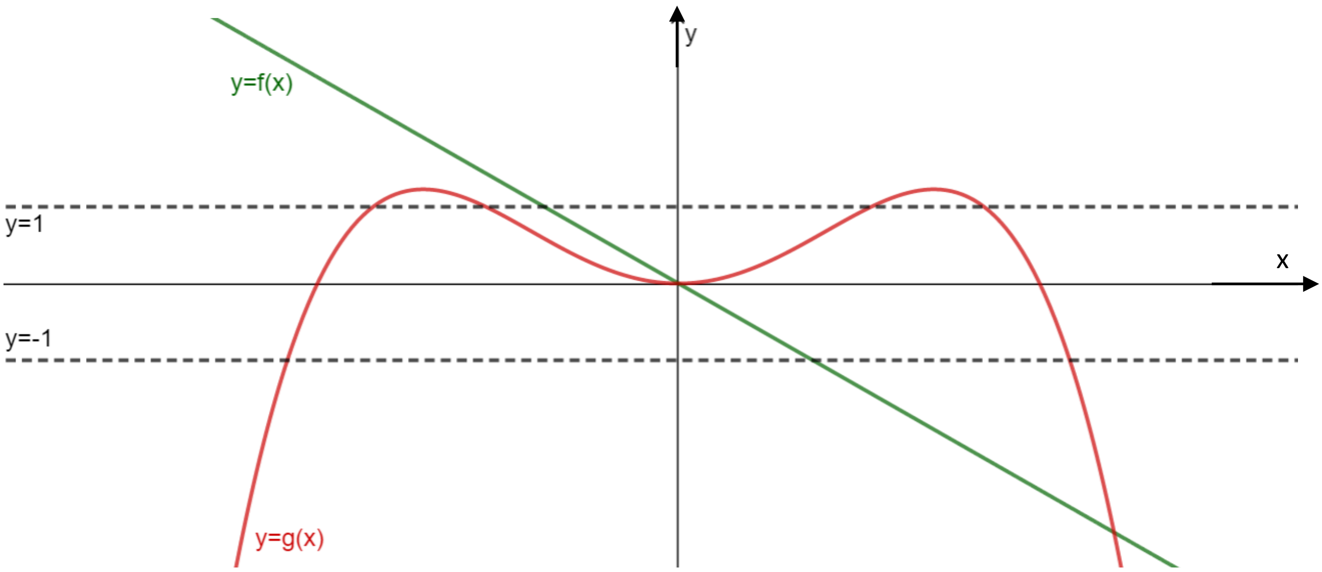


(i) Show that the volume of the right square pyramid is

$$V = \frac{4h^3}{3}\tan^2\alpha.$$ **3**

(ii) A full wheat silo, with the same dimensions as the inverted right square pyramid from (i) has wheat flowing out the bottom of it through a hole at point E. Given h =225cm, $\alpha = 30^\circ$ and grain is flowing out at the rate of $\frac{dV}{dt} = 25\sqrt{h}\, cm^3 / second$, show that $\frac{dh}{dt} = \frac{1}{180} cm / second$. **2**

(b) The graphs of $y=1, y=-1, y=f(x)$ and $y=g(x)$ are shown below



i) Trace a half page copy of the above graph in your answer booklet.

ii) On the same graph, sketch $y = f(x) \times g(x)$ **3**

End of examination

Fort St High School	

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SOLUTIONS- Mathematics Extension 1

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Question 1. **(8 marks)**

(a) Find the cartesian equation represented by the parametric equations

$x = 2 - p$

$y = p^2 + 1$ **2**

Solution

$p = 2 - x$

$y = (2-x)^2 + 1$

$y = 4 - 4x + x^2 + 1$

$y = x^2 - 4x + 5$

Marking Guidelines	
2	Correct response
1	For substituting $p = 2 - x$ into $y = p^2 + 1$
Marker's comments	
Well done.	

(b) $P(x) = x^3 + 2x^2 - 5x + 1$.

(i) Without using polynomial division, show that $P(x) \div (x-2)$ has remainder 7. **1**

Solution

Marking Guidelines	
1	Correct response
Marker's comments	
Well done.	

$P(2) = 2^3 + 2(2)^2 - 5(2) + 1$

$= 7$

$\therefore$ By remainder theorem

$P(x) \div (x-2)$ has remainder 7

(ii) Verify the result of (i) using polynomial division. **3**

Solution

$$
\begin{array}{r}
x^2+4x+8 \\
x-2\ \overline{\big)\ x^3+2x^2-5x+1} \\
\underline{x^3-2x^2} \\
4x^2-5x \\
\underline{4x^2-8x} \\
3x+1 \\
\underline{3x-6} \\
7
\end{array}
$$

Marking Guidelines	
3	Correct response
2	Two correct iterations of division process
1	One correct iteration of division process
Marker's comments	
Well done.	

(c) By using a compound angle formula, show that the exact value of $\tan 105^\circ$ is $-\left(2+\sqrt{3}\right)$ **2**

Solution

$$\tan 105^\circ$$

$$= \tan(60+45)^\circ$$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \times \tan 45^\circ}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}\times 1}$$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{3+2\sqrt{3}+1}{1-3}$$

$$= \frac{4+2\sqrt{3}}{-2}$$

$$= -(2+\sqrt{3})$$

Marking Guidelines	
2	Correct response
1	A correct choice of angles followed by correct expansion of the tan compound angle identity.
Marker's comments	
Mostly well done. Some students lost a mark for not demonstrating how to rationalise the denominator.	

Question 2. (9 marks) *Start a separate Booklet*

(a) Find the domain and range of $y = a\sin^{-1}\left(\frac{x}{b}\right)$, where a and b are constants. **2**

Solution

Domain: $-1 \le \frac{x}{b} \le 1$

$-b \le x \le b$

Range $-\frac{\pi}{2} \le \frac{y}{a} \le \frac{\pi}{2}$

$-\frac{a\pi}{2} \le y \le \frac{2y}{a}$

Marking Guidelines	
2	Correct response
1	Either domain or range correct.
Marker's comments	
This question was poorly answered. Students need to revise the domain and range of the basic inverse trigonometric functions and how to apply these to find the domain and range of complex trigonometric functions.	

(b) $P(x) = 2x^3 - ax^2 + bx + 3$, where a and b are constants. When $P(x)$ is divided by $x^2 - 1$ the remainder is $-2x - 2$. Find a and b. **3**

Solution

By the division algorithm,

$P(x) = (x^2-1)\,Q(x) - 2x - 2$

$= (x-1)(x+1)\,Q(x) - 2(x+1)$

$\therefore P(1) = -4$

$2(1)^3 - a(1)^2 + b(1) + 3 = -4$

$b - a = -9 \quad \ldots (1)$

also

$P(-1) = 0$

$2(-1)^3 - a(-1)^2 + b(-1) + 3 = 0$

$-a - b = -1$

$a + b = 1 \quad \ldots (2)$

Marking Guidelines	
3	Correct response.
2	Attaining both equations (1) and (2)
1	Applying the division algorithm once to derive equation (1) or (2)
Marker's comments	
Poorly answered. Many students have no idea of the division transformation and how it can be used to answer this question. Many students assumed incorrectly that the remainder equalled zero when P(x)is divided by x-1 or x+1. An alternative method is to do divide P(x) by x^2-1, obtain the remainder and equate it to the given remainder. Then equate coefficients to find a and b. If you use this method, do not combine the remainders.	

(1) + (2) $\quad 2b = -8$

$b = -4 \; \ldots (3)$

Sub. (3) in (2)

$a = 5$

(c) (i) Given $f(x) = x^2 - 2x + 2$ for $x \le 1,\ y \ge 1$ what is the domain and range for $f^{-1}(x)$? **1**

Solution

Domain $x \ge 1$,

Range $y \le 1$.

Marking Guidelines	
1	Correct response
Marker's comments	
Well done by most students.	

(ii) By interchanging x and y, show that inverse function of $f(x) = x^2 - 2x + 2$ for $x \le 1,\ y \ge 1$ is

$f^{-1}(x) = 1 - \sqrt{x-1}$ **3**

Solution

$f^{-1}(x)$: $x = y^2 - 2y + 2$

$= y^2 - 2y + 1 \; + 1$

$x = (y-1)^2 + 1$

$x - 1 = (y-1)^2$

$y - 1 = \pm\sqrt{x-1}$

$y = 1 \pm \sqrt{x-1}$

but range is $y \le 1$,

$\therefore\ y = 1 - \sqrt{x-1}$

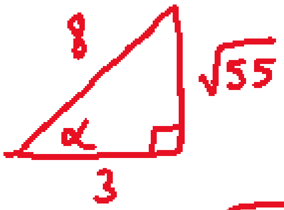
Marking Guidelines	
3	Correct response
2	For obtaining $y = 1 \pm \sqrt{x-1}$
1	For interchanging x and y
Marker's comments	
Most students were able to obtain the first mark. However, many students could not complete the square to make y the subject in the inverse function and lost 2 marks. Also, this is a 'SHOW' question and many students failed to explain there are two cases for y and that only one of them is the answer because of the restricted range.	

Question 3. **(8 marks)** *Start a separate Booklet*

(a) By letting $\alpha = \cos^{-1}\left(\frac{3}{8}\right)$ show, using a right triangle and a double angle formula, that

$\sin\left(2\cos^{-1}\left(\frac{3}{8}\right)\right)$ has exact value $\frac{3\sqrt{55}}{32}$. **3**

<u>Solution</u>



$\therefore \sin\alpha = \frac{\sqrt{55}}{8}$

$\therefore \sin(2\alpha) = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{\sqrt{55}}{8} \times \frac{3}{8}$

$= \frac{3\sqrt{55}}{32}$

Marking Guidelines	
3	Correct response
2	Correct substitution into formula for $\sin 2\alpha$
1	Finding $\sin\alpha = \frac{\sqrt{55}}{8}$
Marker's comments	
Mostly well done .	

(b) (i) Use the t-results to show that $\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \tan\dfrac{\theta}{2}$ **3**

<u>Solution</u>

$$\text{LHS} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}}$$

$$= \sqrt{\frac{1+t^2-(1-t^2)}{1+t^2+1-t^2}}$$

$$= \sqrt{\frac{2t^2}{2}}$$

$$= t$$

$$= \tan\theta/2$$

$$= \text{RHS.}$$

Marking Guidelines	
3	Correct response
2	Correct simplification to obtain $\sqrt{\dfrac{2t^2}{2}}$
1	Correct use of $\cos\theta = \dfrac{1-t^2}{1+t^2}$ to substitute into LHS.
Marker's comments	
Generally answered well . A number of students made mistakes in dealing with algebraic fractions . They should practice on their algebraic skills .	

(ii) Hence, show that $\sqrt{3+2\sqrt{2}} = \tan 67\cdot 5^\circ$ **2**

Solution

Let $\theta = 135^\circ$.

Using $\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$$\tan\left(\frac{135}{2}\right) = \sqrt{\frac{1-\cos 135^\circ}{1+\cos 135^\circ}}$$

$$= \sqrt{\frac{1 - -\frac{1}{\sqrt{2}}}{1 + -\frac{1}{\sqrt{2}}}}$$

$$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}}$$

$$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}} \div \frac{\sqrt{2}-1}{\sqrt{2}}}$$

$$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}-1}}$$

$$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$$

$$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}}$$

$$= \sqrt{\frac{2+2\sqrt{2}+1}{2-1}}$$

$$= \sqrt{3+2\sqrt{2}}$$

Marking Guidelines	
2	Correct response
1	Letting $\theta = 135^\circ$

Marker's comments

Not well done . Most of the students couldn't identify that the angle theta should be 135 degrees . Some students didn't even use (i) to do part (ii). They should be aware of the meaning of hence. They used other methods and some just used calculators to show that the left hand side is equal to the right hand side and received no mark . They should read the question carefully.

Question 4. (9 marks) *Start a separate Booklet*

(a) If $f(x)=\ln(ex)$ and $g(x)=e^{x-1}$ show that

(i) $f(g(x))=x$ **2**

Solution

$$f(g(x)) = f(e^{x-1})$$
$$= \ln(e \times e^{x-1})$$
$$= \ln(e^x)$$
$$= x$$

Marking Guidelines	
2	Correct response
1	for attaining $\ln(e \times e^{x-1})$
Marker's comments	
Students that did not receive full marks should review their index laws	

(ii) $g(f(x))=x$ **2**

Solution

$$g(f(x)) = g(\ln(ex))$$
$$= e^{\ln(ex)-1}$$
$$= \frac{e^{\ln(ex)}}{e^1}$$
$$= \frac{ex}{e}$$
$$= x$$

Marking Guidelines	
2	Correct response
1	for attaining $e^{\ln(ex)-1}$
Marker's comments	
Similarly to part (i) some students should review index laws.	

(iii) What do the results of (i) and (ii) indicate about $f(x)$ and $g(x)$? **1**

Solution

They are mutually inverse functions.

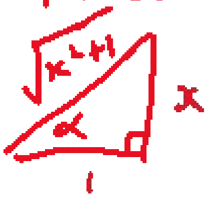
Marking Guidelines	
1	Correct response
Marker's comments	
Many students communicated their solutions poorly and in some cases with one word answers such as "inverse". Many students came close to losing a mark. A HSC marker would **not** likely award the mark to many answers It does not take much to write a grammatically correct sentence that communicates mathematical information.	

(b) (i) Show that $\sin\left(\tan^{-1} x\right) = \dfrac{x}{\sqrt{x^2+1}}$ **2**

Solution

Let $\alpha = \tan^{-1} x$

$\tan \alpha = x$



$\therefore \sin(\tan^{-1} x) = \sin \alpha$

$= \dfrac{x}{\sqrt{x^2+1}}$

Marking Guidelines	
2	Correct response
1	Partial solution including full diagram and introduction of α .

Marker's comments

Many students have **not** grasped how to set out a 'show that' solution. You are required to be explicit.

Many diagrams did not contain the reference angle α or communicate to the examiner that $\alpha = \tan^{-1} x$. This is basic communication. The examiner is not allowed to interpret your intention. You are communicating it.

Many students worked across the page (i.e. xxx = xxx = xxx = xxx). This is not the convention.

(ii) Hence or otherwise, show that $f(x) = \sin\left(\tan^{-1} x\right)$ is an odd function. **2**

Solution

Either

$f(x) = \sin\left(\tan^{-1} x\right)$

$= \dfrac{x}{\sqrt{x^2+1}}$

$\therefore f(-x) = \dfrac{-x}{\sqrt{(-x)^2+1}}$

$= \dfrac{-x}{\sqrt{x^2+1}}$

$= -f(x) \quad \therefore$ odd

Marking Guidelines	
2	Correct response
1	Correct substitution of -x into f(x).

Marker's comments

Again, many students have **not** grasped how to set out a 'show that' solution. You are required to be explicit and show course concepts.

Examiners should not have to fill in missing steps. You should lay it out for the marker. It is expected you show **all** steps, no matter how trivial you might feel they are. For example many students were going from

$$f(-x) = \sin\left(\tan^{-1}(-x)\right)$$
$$= -\sin\left(\tan^{-1} x\right)$$

without communicating the properties of odd functions (see solution).

or $f(x) = \sin\left(\tan^{-1}(x)\right)$

$f(-x) = \sin\left(\tan^{-1}(-x)\right)$

$= \sin\left(-\tan^{-1} x\right)$ as $\tan^{-1} x$ is odd

$= -\sin\left(\tan^{-1} x\right)$ as $\sin x$ is odd

$= -f(x)$

$\therefore f(x)$ is odd.

Question 5. (9 marks) *Start a separate Booklet*

(a) Prove that $\dfrac{x-2}{5x+1} \geq \dfrac{3}{4}$ has solutions $-1 \leq x < -\dfrac{1}{5}$. **3**

Solution

$$\frac{x-2}{5x+1} \geq \frac{3}{4} \qquad \times 4(5x+1)^2$$

$$4(x-2)(5x+1) \geq 3(5x+1)^2$$

$$4(x-2)(5x+1) - 3(5x+1)^2 \geq 0$$

$$(5x+1)\left[4(x-2) - 3(5x+1)\right] \geq 0$$

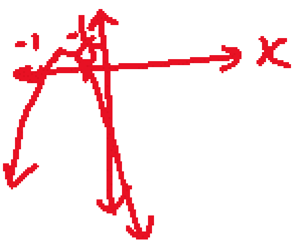
$$(5x+1)(-11 - 11x) \geq 0$$

$$-11(5x+1)(1+x) \geq 0$$

$$-1 \leq x < -\frac{1}{5}$$

as $x \neq -1/5$

due to division by zero error.



Marking Guidelines	
3	Correct response
2	Simplifying to a factorised quadratic inequality
1	Multiplying by the square of the denominator or equivalent merit.

Marker's comments

- A mark was deducted if students didn't state that $x \neq -\frac{1}{5}$
- Some students didn't multiply both sides by $(5x+1)^2$ and consequently lost a mark
- Careless errors made when students expanded after multiplying both sides by $(5x+1)^2$ instead of factorising and when working with fractions instead of multiplying by 4.

(b) Frozen meat is taken out of a freezer by a chef at $-18^\circ C$, placed in a room with temperature $25^\circ C$ and allowed to defrost. The meat warms according to the law

$$\frac{dT}{dt} = k\left(T_o - T\right)$$

where T is the temperature of the meat, T_o the room temperature, t the time in minutes since meat is taken from the freezer and k is a constant.

(i) Show that $T = 25 - 43e^{-kt}$ is a solution of the differential equation $\dfrac{dT}{dt} = k\left(T_o - T\right)$. **2**

Solution

$$LHS = \frac{dT}{dt}$$

$$= \frac{d}{dt}\left(25 - 43e^{-kt}\right)$$

$$= 43ke^{-kt}$$

$$RHS = k(25 - T)$$

Marking Guidelines	
2	Correct response
1	

Marker's comments

- This question was not answered very well as most students did not set out the work as a formal proof
- A mark was deducted when students undertook algebraic manipulation of $\frac{dT}{dt}$ instead f substituting T into the RHS

$$= k(25-(25-43e^{-kt}))$$

$$RHS = k(43e^{-kt})$$

$$= 43ke^{-kt}$$

$$= LHS$$

(ii) After 30 minutes, the chef measures the temperature of the meat to be $-5^\circ C$. Show that the exact value of k is $\dfrac{-1}{30}\ln\left(\dfrac{30}{43}\right)$. **2**

Solution

When $t=30$, $T=-5$

$$\therefore -5 = 25-43e^{-30k}$$

$$-30 = -43e^{-30k}$$

$$\frac{30}{43} = e^{-30k}$$

$$\ln\left(\frac{30}{43}\right) = -30k$$

$$k = -\frac{1}{30}\ln\left(\frac{30}{43}\right)$$

Marking Guidelines	
2	Correct response
1	Obtaining $-30=-43e^{-30k}$
Marker's comments	
• Generally answered well	

(iii) Find the time t that allows the chef to begin cooking the meat once it reaches a temperature of $10^\circ C$. Give your answer to the nearest minute. **2**

Solution

$$10 = 25-43e^{-kt}$$

$$-15 = -43e^{-kt}$$

$$\frac{15}{43} = e^{-kt}$$

$$\ln\left(\frac{15}{43}\right) = -kt$$

$$t = -\frac{1}{k}\ln\left(\frac{15}{43}\right)$$

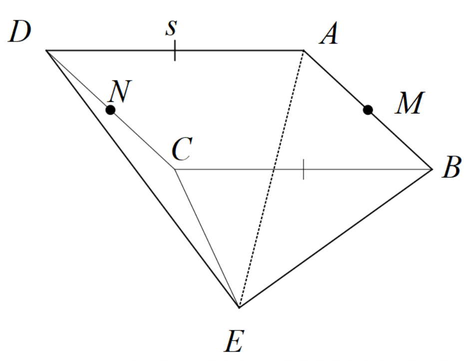
$$= \frac{30}{\ln\left(\frac{30}{43}\right)} \times \ln\left(\frac{15}{43}\right)$$

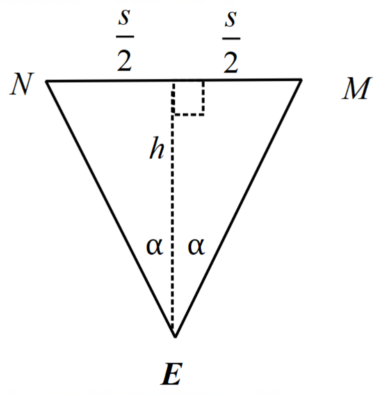
Marking Guidelines	
2	Correct response
1	
Marker's comments	
• Generally answered well	

$= 88$ minutes
(nearest minute)

Question 6. **(8 marks)** *Start a separate Booklet*

(a) An inverted right square pyramid ABCDE with square sidelength s and midpoints M and N on sides AB and CD, is shown below left. The cross section of the pyramid through N, M and E with height h and semi-apex angle α is shown below right.





(i) Show that the volume of the right square pyramid is

$$V = \frac{4h^3}{3}\tan^2\alpha.$$

3

Solution

$V = \frac{1}{3} A H$

$= \frac{1}{3} s^2 h$

but $\tan\alpha = \frac{s}{2h}$

$2h\tan\alpha = s$

$\therefore V = \frac{1}{3}(2h\tan\alpha)^2 h$

$V = \frac{4h^3\tan^2\alpha}{3}$

Marking Guidelines	
3	Correct response
2	Correct solution up to the Substitution of $s = 2h\tan\alpha$ into volume formula or equivalent merit.
1	Stating $V = \frac{1}{3}s^2 h$
Marker's comments	
Mostly done well. You cannot skip steps in a "Show" question. This was the most common reason for any loss of marks.	

(ii) A full wheat silo, with the same dimensions as the inverted right square pyramid from (i) has wheat flowing out the bottom of it through a hole at point E. Given h =225cm, $\alpha = 30^\circ$ and grain is flowing out at the rate of $\frac{dV}{dt} = 25\sqrt{h}\ cm^3 / second$, show that $\frac{dh}{dt} = \frac{1}{180} cm / second$. **2**

Solution

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

where $V = \frac{4}{3} h^3 \tan^2 \alpha$

$$\frac{dV}{dh} = 4h^2 \tan^2 \alpha$$

when $h = 225\text{cm}$, $\alpha = 30°$

$$\frac{dV}{dh} = 4 \times 225^2 \times (\tan 30°)^2$$

$$= 4 \times 225^2 \times \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 67500$$

$$\frac{dV}{dt} = 25\sqrt{225}$$

$$= 375$$

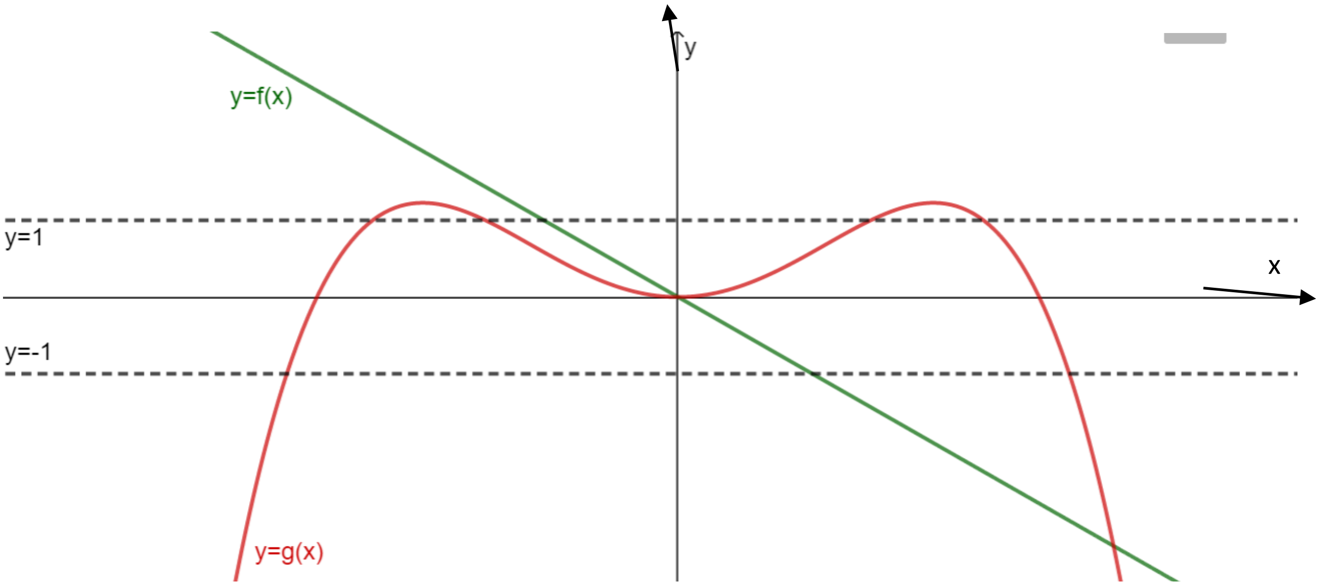
$$\therefore \frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$= \frac{1}{67500} \times 375$$

$$= \frac{1}{180} \text{ cm/s}$$

Marking Guidelines	
2	Correct response
1	Showing that $\frac{dV}{dh} = 67500$ or equivalent merit
Marker's comments	
Generally answered well.	

(b) The graphs of $y=1, y=-1, y=f(x)$ and $y=g(x)$ are shown below



i) Trace a half page copy of the above graph in your answer booklet.

ii) On the same graph, sketch $y=f(x)\times g(x)$ **3**

Marking Guidelines	
3	Correct response
2	Only 1 error
1	Partially correct graph.

Marker's comments

Very few students received full marks in this question. To get 3/3 you needed to correctly sketch x-intercepts, align the curve with known points $y=1$ and $y=-1$ and also get all aspects of the shape correct. This includes the steepness of the curve as $x\to 0$. Notice in the solution that the black curve approaches the origin from the left "below" the red curve. You know this to be the case since $f(x)<1$ and $g(x)<1$ which means $f(x)\times g(x) < g(x)$ on this part of the graph.

It seemed that many students knew how to do the question but did not take enough care with their sketching. Curve sketching is an important skill and it needs to have an appropriate level of detail, especially in a 3 mark question.

<u>Solution</u>