Name: ....

Teacher: ....

Fort Street High School

2020

Preliminary HSC Assessment Task 3

Mathematics Extension 1

Syllabus Outcomes	Assessment Area
ME11-1	Uses algebraic and graphical concepts in the modelling and solving of problems involving functions
ME11-2	Manipulates algebraic expressions and graphical functions to solve problems
ME11-3	Applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems
ME11-4	Applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change
ME11-7	Communicates making comprehensive use of mathematical language, notation, diagrams and graphs

Time allowed: 90 minutes (+ 10minutes reading time)

General Instructions:

- Marks may be deducted for careless or poorly arranged work
- NESA-approved calculators may be used
- A Reference Sheet is provided with this paper
- Use a separate booklet for each question

Question	Mark
MCQ	**/7**
8	**/13**
9	**/14**
10	**/13**
11	**/13**
Total	**/60**
	%

Section I   7 marks

Attempt Questions 1–7

Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 7.

1. Simplify $\sin(A+B)\cos A - \cos(A+B)\sin A$

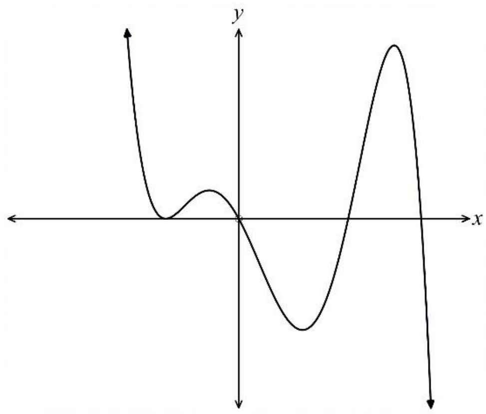
(A) $\sin B$

(B) $\cos B$

(C) $\sin(2A+B)$

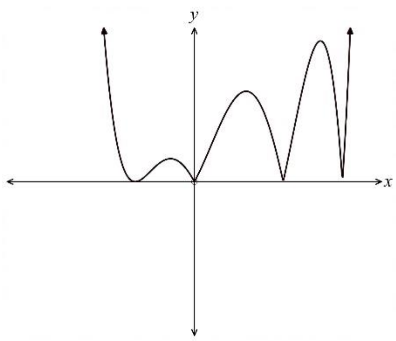
(D) $\cos(2A+B)$

2. The graph of $y = f(x)$ is shown below.

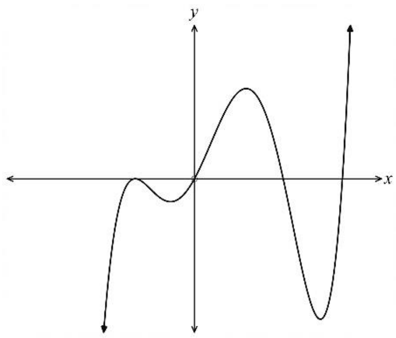


Which graph shows $= f(|x|)$?

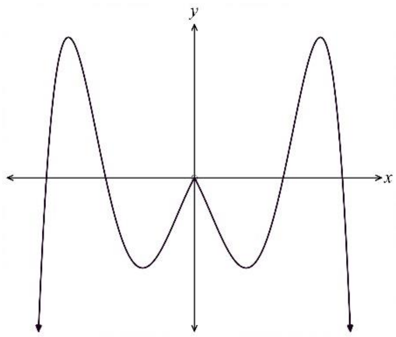
(A)



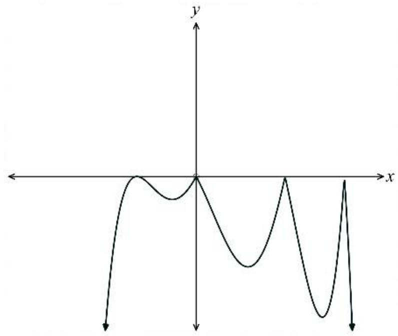
(B)



(C)



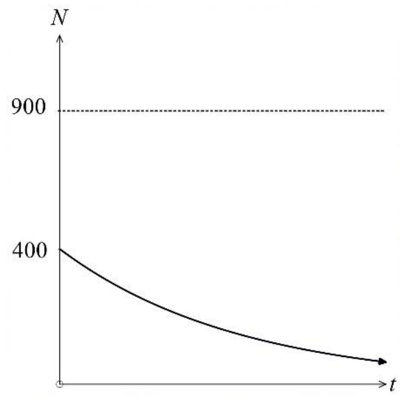
(D)



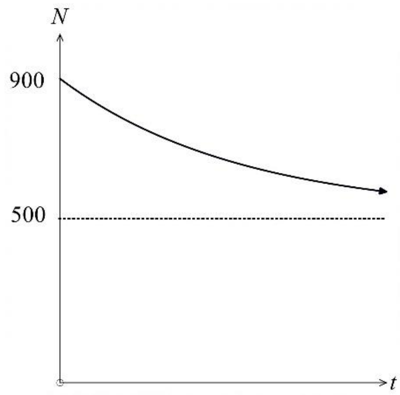
3. A quantity N has an initial value of 900 and the rate of change of N is given by the equation $\frac{dN}{dt} = 0.35(N - 500)$.

Which graph shows the relationship between N and t ?

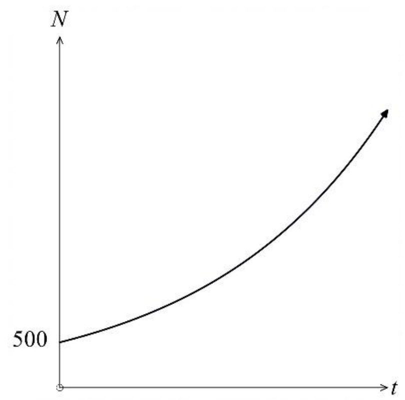
(A)



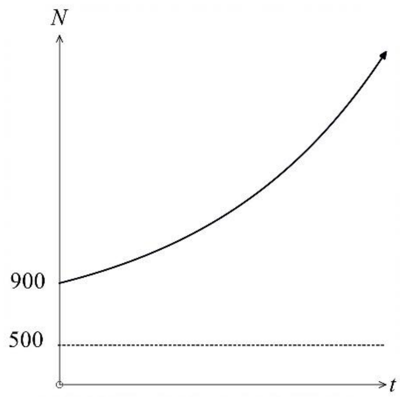
(B)



(C)

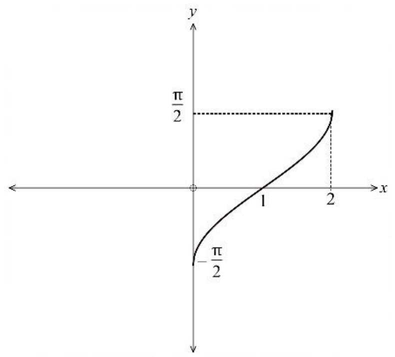


(D)

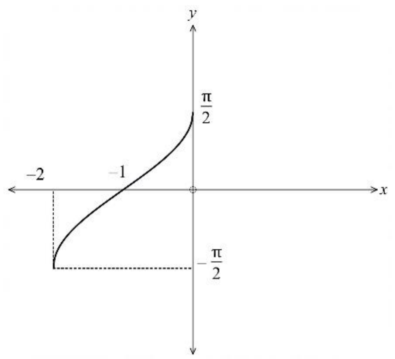


4. Which graph shows the curve $y = \sin^{-1}(x + 1)$?

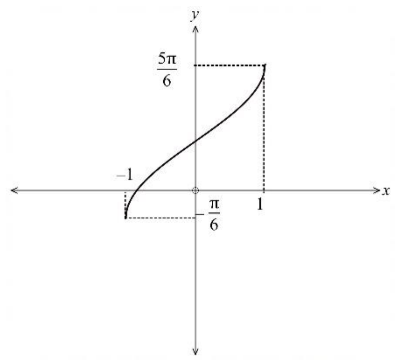
(A)



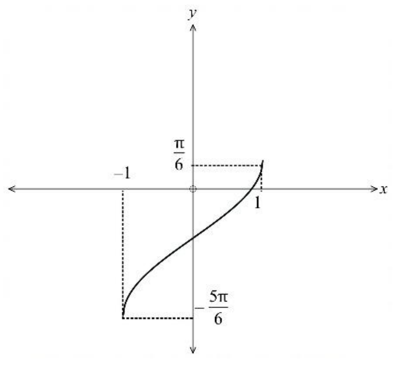
(B)



(C)



(D)



5. The diameter of a circle is increasing at the rate of 1 $cm/\sec$

At what rate is the area increasing when its radius is π cm ?

(A) $\pi \quad cm^2/\sec$

(B) $2\pi \quad cm^2/\sec$

(C) $\pi^2 \quad cm^2/\sec$

(D) $2\pi^2 \quad cm^2/\sec$

6. Let $P(x)=ax^3+bx^2+bx+a$ where a and b are constants , $a\neq 0$. One of the zeros of $P(x)$ is -1 .

Given that α is a zero of $P(x)$, $\alpha\neq -1$,which of the following is also a zero?

(A) $-\dfrac{1}{\alpha}$

(B) $-\dfrac{a}{\alpha}$

(C) $\dfrac{1}{\alpha}$

(D) $\dfrac{a}{\alpha}$

7. What is the exact value of

$$\left(\tan\frac{\pi}{180}\right)\times\left(\tan\frac{2\pi}{180}\right)\times\left(\tan\frac{3\pi}{180}\right)\times\left(\tan\frac{4\pi}{180}\right)\times\text{.........................}\times\left(\tan\frac{88\pi}{180}\right)\times\left(\tan\frac{89\pi}{180}\right)$$

(A) 1

(B) 0

(C) -1

(D) Not defined

Mathematics - Extension 1

Section II

53 marks

Attempt Questions 8 – 11

Allow about 1 hour and 20 minutes for this section

Instructions

- Answer each question in the separate writing booklet.
- Extra writing booklets are available.
- In Questions 8–11, your responses should include relevant mathematical reasoning and/ or calculations.

Section II

53 marks

Attempt Questions 8 – 11.

Allow about 1 hour and 20 minutes for this section.

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 8 – 11, your responses should include relevant mathematical reasoning and/or calculations.

Question 8 (13 marks) Use the separate writing booklet.

(a) Solve the inequality $\frac{3}{x-1} \leq 2$ **3**

(b) A monic polynomial $P(x)$ of degree 4 has one zero equal to 2 and a double zero equal to -1. If the polynomial passes through the point $(0,6)$, Find the equation of $P(x)$. **3**

(c) A curve has parametric equations $x = 4\sin t + 3$ and $y = 4\cos t - 1$.

(i) Eliminate the parameter t and show that the curve is a circle and give its Cartesian equation. **2**

(ii) Find the *exact* co-ordinates of the point on the curve where $t = \frac{\pi}{4}$. **2**

(d) Given the function , $f(x) = \frac{e^{2x}+1}{2e^{x}}$. **3**

Determine if the function is odd, even or neither by showing all of your working out.

Question 9 (14 marks) Use the separate writing booklet.

(a) Evaluate the expression $\tan\left(\sin^{-1}\left(-\frac{1}{5}\right)\right)$. **3**

Give your answer in the form of $a\sqrt{b}$ where a and b are rational.

(b) The length of a rectangle remains twice as its width at all times when the rectangle is expanding. The area of the rectangle is increasing at a rate of $16\ cm^2/\sec$. Find the rate at which the perimeter of the rectangle is increasing at the instant when its width is $200\ cm$. **3**

(c) The cubic equation $2x^3 - 3x + 2 = 0$ has three real roots α , β and γ .

Evaluate :

(i) $\alpha^3\beta^3\gamma^2 + \alpha^3\beta^2\gamma^3 + \alpha^2\beta^3\gamma^3$ **2**

(ii) $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta}$ **3**

(d) Using the t formula , solve $\sin x = 3\cos x + 2$, for $0^\circ \le x \le 360^\circ$ to the nearest degree. **3**

Question 10 (13 marks) Use the separate writing booklet.

(a) $P(x) = x^4 + 7x^3 + 9x^2 - 27x + c$ has a zero of multiplicity 3. **3**

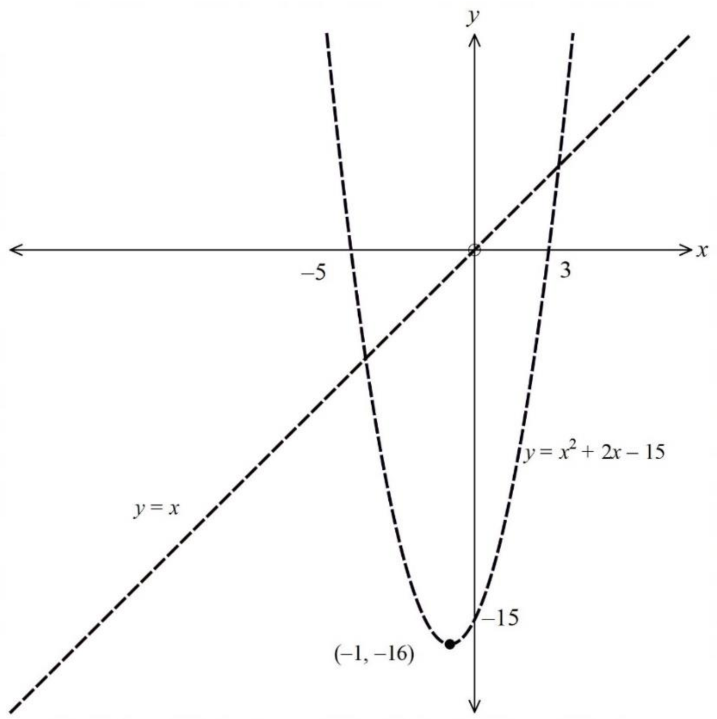
Find the value of c .

(b) If α and β are acute angles such that $\tan\alpha = \dfrac{m}{m+1}$ and $\tan\beta = \dfrac{1}{2m+1}$. **3**

Prove that $\alpha + \beta = \dfrac{\pi}{4}$

(c) A function is defined as $f(x) = x^2 + 2x - 15$.

The graphs of $y = f(x)$ and of the line $y = x$ are shown on the diagram below.



(i) Determine what restriction including its y - intercept would need to be put on the domain of $f(x)$ if it is to have an inverse function $f^{-1}(x)$. **1**

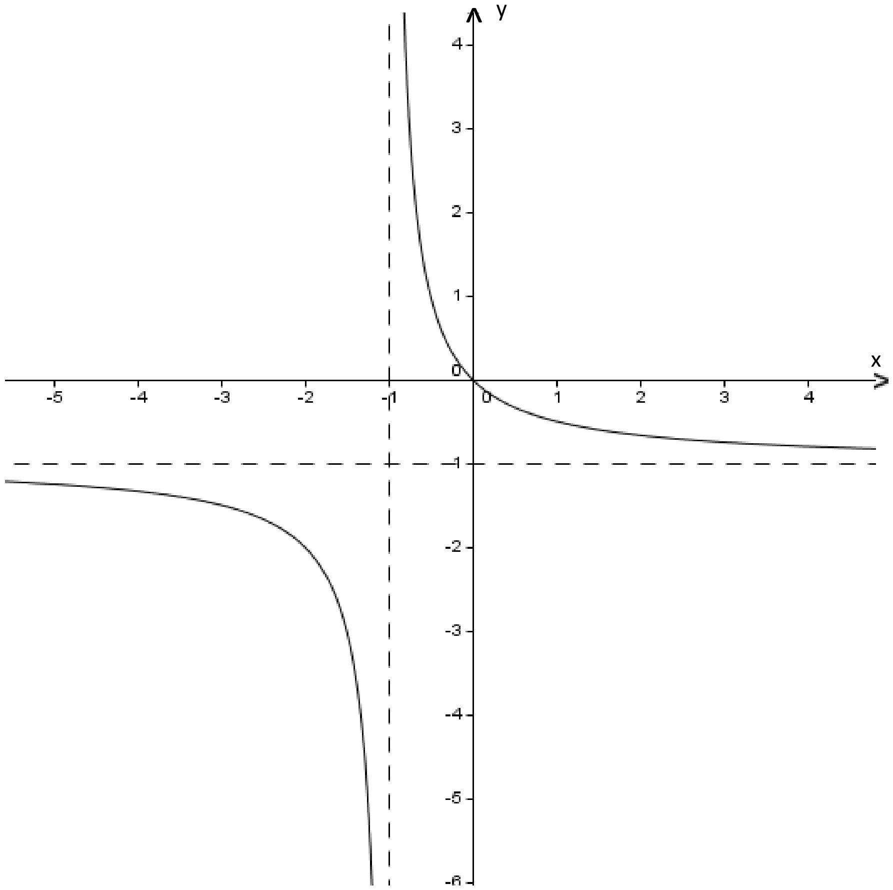
(ii) Determine the equation of the inverse function $f^{-1}(x)$. **2**

(iii) Draw a sketch showing the graph of $y = f^{-1}(x)$. **1**

(d) Given $\sec x + \tan x = k$, Find the value of $\cos x$ in terms of k. **3**

Question 11 (13 marks) Use the separate writing booklet.

(a) The diagram below is a sketch of the graph of the function $f(x) = -\dfrac{x}{x+1}$



(i) Sketch the graph of $y = (f(x))^2$, showing all the asymptotes and intercepts. **2**

(ii) Sketch the graph of $y = x + f(x)$,showing all asymptotes and intercepts. **2**

(iii) Solve the equation $(f(x))^2 = f(x)$ **1**

(b) In a group of 1000 computers linked to each other via the internet, the number N infected with a virus at time t years is given by

$$N = \frac{1000}{1 + ce^{-1000t}}$$

Where c is a constant.

(i) Show that , eventually , all the computers will be infected with the virus. **1**

(ii) At the beginning ,only one computer was infected with the virus. **2**

After how many days will the 50% of the computers be infected?

(iii) Show that $\frac{dN}{dt} = N(1000 - N)$ **2**

(c) The table shows selected values of a one-to-one differential function $g(x)$ **3**

and its derivative $g'(x)$.

x	-1	0
$g(x)$	-5	-1
$g'(x)$	3	$\frac{1}{2}$

Let $f(x)$ be a function such that $f(x) = g^{-1}(x)$.

Find the value of $f'(-1)$.

Justify your answer with mathematical reasoning.

End of Paper

Year 11 Final Examination 2020

Mathematics Extension 1 Course

Name ______________________ Teacher ____________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct ↙) C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○

Name: ....

Teacher: ....

Fort Street High School

2020

Preliminary HSC Assessment Task 3

Mathematics Extension 1

SOLUTIONS

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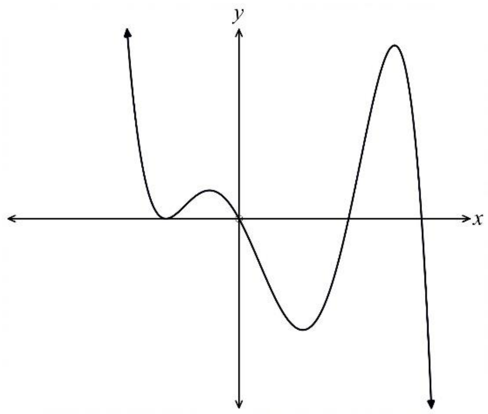

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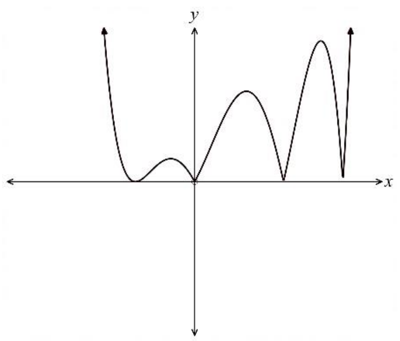
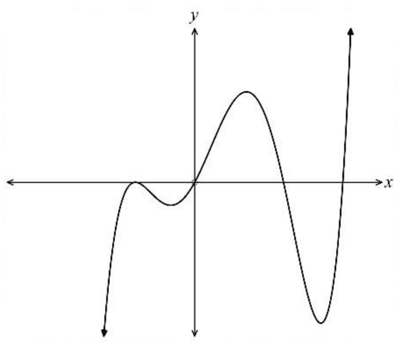
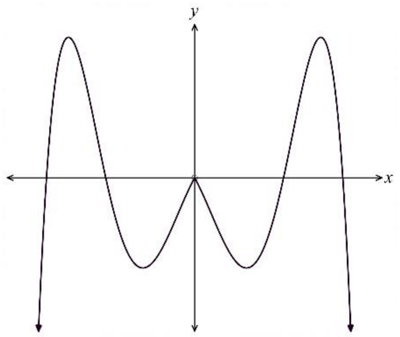


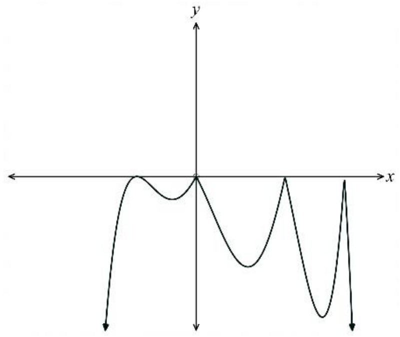
(B)



(C)



(D)



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Which graph shows the relationship between N and t ?

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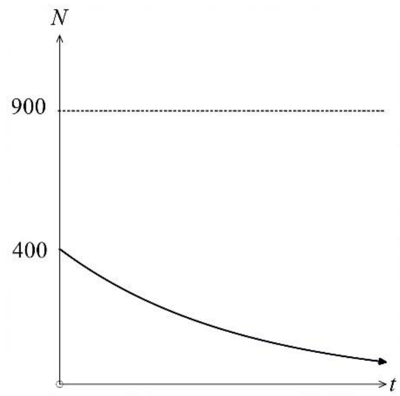

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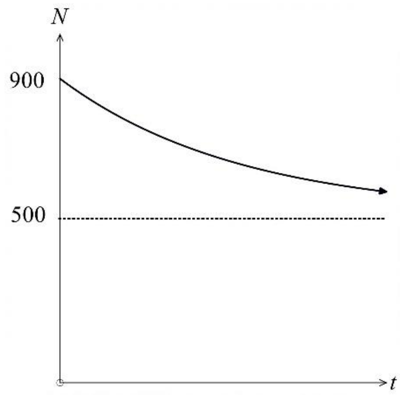
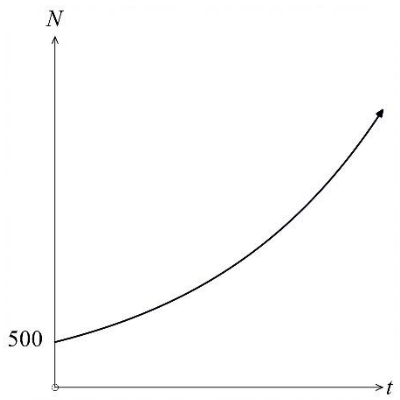


(C)



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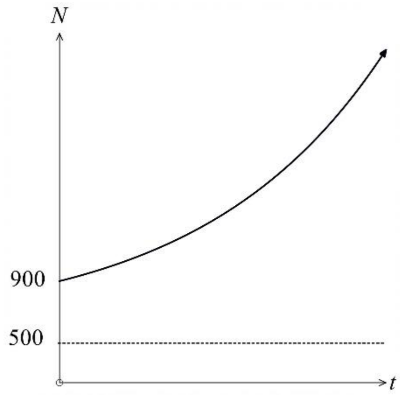
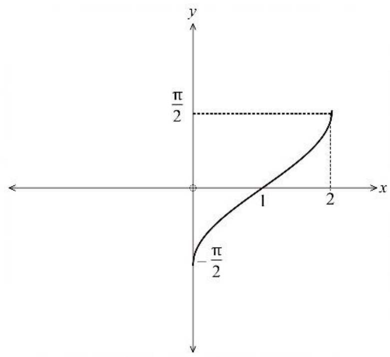
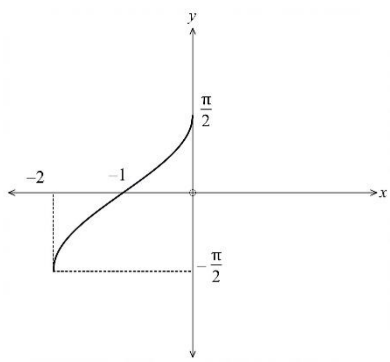


4. Which graph shows the curve $y = \sin^{-1}(x + 1)$?

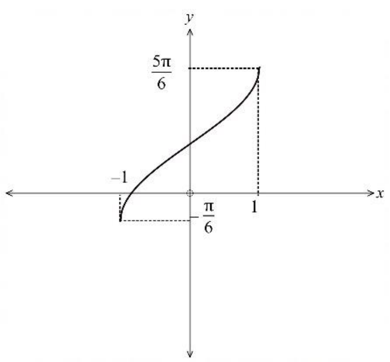
(A)



(B)



(C)



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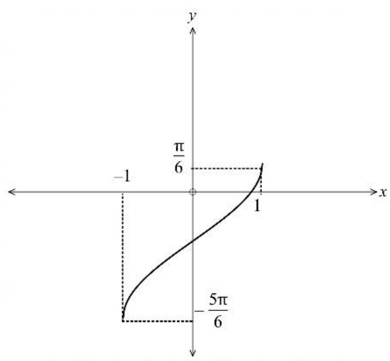


5. The diameter of a circle is increasing at the rate of 1 $cm/\sec$

At what rate is the area increasing when its radius is π cm ?

(A) $\pi \quad cm^2/\sec$

(B) $2\pi \quad cm^2/\sec$

(C) $\pi^2 \quad cm^2/\sec$

(D) $2\pi^2 \quad cm^2/\sec$

6. Let $P(x)=ax^3+bx^2+bx+a$ where a and b are constants , $a\neq 0$. One of the zeros of $P(x)$ is -1 .

Given that α is a zero of $P(x)$, $\alpha\neq -1$,which of the following is also a zero?

(A) $-\dfrac{1}{\alpha}$

(B) $-\dfrac{a}{\alpha}$

(C) $\dfrac{1}{\alpha}$

(D) $\dfrac{a}{\alpha}$

7. What is the exact value of

$$\left(\tan\frac{\pi}{180}\right)\times\left(\tan\frac{2\pi}{180}\right)\times\left(\tan\frac{3\pi}{180}\right)\times\left(\tan\frac{4\pi}{180}\right)\times\text{.........................}\times\left(\tan\frac{88\pi}{180}\right)\times\left(\tan\frac{89\pi}{180}\right)$$

(A) 1

(B) 0

(C) -1

(D) Not defined

Multiple Choice Questions

	A	B	C	D
1.	●	○	○	○
2.	○	○	●	○
3.	○	○	○	●
4.	○	●	○	○
5.	○	○	●	○
6.	○	○	●	○
7.	●	○	○	○

Detailed solutions of Multiple Choice Questions

Q1.

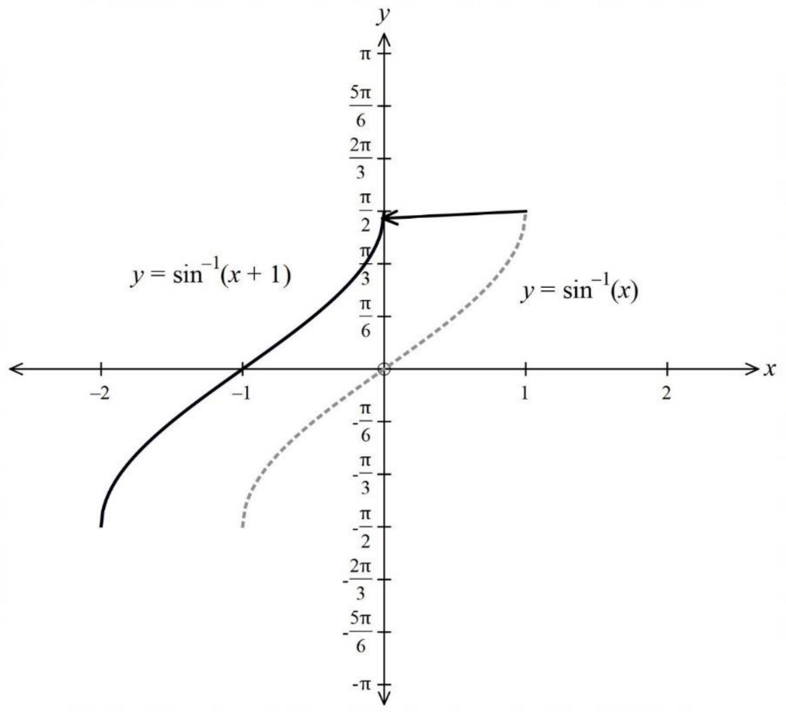
$\sin(A+B)\cos A - \cos(A+B)\sin A$
$= \sin(A+B-A)$
$= \sin B$

Q2.

$y = f(|x|)$ takes all negative values of x and
makes them positive before evaluating $f(x)$
So $f(-a) = f(a)$, so for $x < 0$ the graph is a reflection
of the positive section of $f(x)$ in the $y -$ axis

Q3.
A solution to $\frac{dN}{dt} = 0.35(N - 500)$ is $N = 500 + Ae^{0.35t}$
So, the curve is an increasing exponential equation.
When t = 0, N = 900 so the graph starts from 900 on the vertical axis

Q4.



Q5.

$$2 \times \frac{dr}{dt} = 1$$

$$\frac{dr}{dt} = \frac{1}{2}$$

$$A = \pi r^2$$

$$\frac{dA}{dt} = \pi \times 2r \times \frac{dr}{dt} = \pi \times 2 \times \pi \times \frac{1}{2} = \pi^2 cm^2 / \sec$$

Q6. Let β is the third zero .

Then product of the roots

$$-1 \times \alpha \times \beta = -\frac{a}{a} = -1$$

$$\alpha\beta = 1$$

$$\beta = \frac{1}{\alpha}$$

Q7.

$$\left(\tan\frac{\pi}{180}\right)\times\left(\tan\frac{2\pi}{180}\right)\times\left(\tan\frac{3\pi}{180}\right)\times\left(\tan\frac{4\pi}{180}\right)\times\dots\dots\dots\dots\times\left(\tan\frac{45\pi}{180}\right)\times\dots\dots\dots\dots\times\left(\tan\frac{88\pi}{180}\right)\times\left(\tan\frac{89\pi}{180}\right)$$

$$=\left(\tan\frac{\pi}{180}\right)\times\left(\tan\frac{2\pi}{180}\right)\times\left(\tan\frac{3\pi}{180}\right)\times\left(\tan\frac{4\pi}{180}\right)\times\dots\dots\dots\times\left(\tan\frac{45\pi}{180}\right)\times\dots\dots\dots\dots\times\left(\tan\frac{90\pi-2\pi}{180}\right)\times\left(\tan\frac{90\pi-\pi}{180}\right)$$

$$=\left(\tan\frac{\pi}{180}\right)\times\left(\tan\frac{2\pi}{180}\right)\times\left(\tan\frac{3\pi}{180}\right)\times\left(\tan\frac{4\pi}{180}\right)\times\dots\dots\dots\times\left(\tan\frac{45\pi}{180}\right)\times\dots\dots\dots\dots\times\left(\tan\left(\frac{\pi}{2}-\frac{2\pi}{180}\right)\right)\times\left(\tan\left(\frac{\pi}{2}-\frac{\pi}{180}\right)\right)$$

$$=\left(\tan\frac{\pi}{180}\right)\times\left(\tan\frac{2\pi}{180}\right)\times\left(\tan\frac{3\pi}{180}\right)\times\left(\tan\frac{4\pi}{180}\right)\times\dots\dots\dots\dots\times\left(\tan\frac{45\pi}{180}\right)\times\dots\dots\dots\dots\times\left(\cot\frac{2\pi}{180}\right)\times\left(\cot\frac{\pi}{180}\right)$$

$$=\left(\tan\frac{45\pi}{180}\right)=1$$

Question 8 (13 marks) Use the separate writing booklet.

(a) Solve the inequality $\frac{3}{x-1} \le 2$ **3**

Solution:

$$\frac{3}{(x-1)} \times (x-1)^2 \le 2 \times (x-1)^2$$
$$3(x-1) \le 2(x^2 - 2x + 1)$$
$$2x^2 - 7x + 5 \ge 0$$
$$(2x-5)(x-1) \ge 0$$
$$x \ge \frac{5}{2}$$

Or $x \le 1$

But $x \ne 1$

Therefore $x \ge \frac{5}{2}$ or $x < 1$

Marking guidelines

3 Correct solution

2 Correct critical points but did not recognise that $x \ne 1$

1 One correct solution

Markers comments

Many students could not solve the inequality. Students would be well advised to learn this fundamental skill.

(b) A monic polynomial $P(x)$ of degree 4 has one zero equal to 2 and a double zero equal to -1 . If the polynomial passes through the point $(0,6)$, Find the equation of $P(x)$. **3**

Solution:

Let α be the fourth zero of the given polynomial.

$$P(x) = (x-2)(x+1)^2(x-\alpha)$$

Since the given polynomial passes through (0,6)

$$6 = (0-2)(0+1)^2(0-\alpha)$$
$$6 = 2\alpha$$
$$\alpha = 3$$
$$\therefore P(x) = (x-2)(x+1)^2(x-3)$$

Marking guidelines

3 Correct solution

2 Significant progress towards answer regardless of method used.

1 For using $P'(x)$ correctly

Markers comments

Many students made the question harder than it needed to be by considering $P'(x)$

Many students did not read the question closely and use the information that a *monic* polynomial implied the leading coefficient equals one.

Many students did not think to write $P(x)$ out as a product of factors.

(c) A curve has parametric equations $x = 4\sin t + 3$ and $y = 4\cos t - 1$.

(i) Eliminate the parameter t and show that the curve is a circle and give its Cartesian equation. **2**

Solution:

$\sin t = \frac{x-3}{4}$, $\cos t = \frac{y+1}{4}$

$\sin^2 t + \cos^2 t = 1$

As we know $\therefore \left(\frac{x-3}{4}\right)^2 + \left(\frac{y+1}{4}\right)^2 = 1$

$(x-3)^2 + (y+1)^2 = 16$

Marking guidelines

2 Correct solution

1 Partial solution

Markers comments

Many students could not eliminate the parameter by squaring and applying the Pythagorean identity.

Some students will need to revise their understanding of trigonometry as you can not make t the subject by dividing through by sine. E.g. if $4\sin t = x-3$

$t \neq \frac{x-3}{4\sin}$

(ii) Find the *exact* co-ordinates of the point on the curve where $t = \frac{\pi}{4}$. **2**

Solution :

$x = 4\sin\frac{\pi}{4} + 3$

$x = 4 \times \frac{1}{\sqrt{2}} + 3$

$x = 2\sqrt{2} + 3$

$y = 4\cos\frac{\pi}{4} - 1$

$y = 4 \times \frac{1}{\sqrt{2}} - 1$

$y = 2\sqrt{2} - 1$

$\therefore x = 2\sqrt{2} + 3, y = 2\sqrt{2} - 1$

Marking guidelines

2 Correct solution

1 Partial solution

Markers comments

Many students overlooked that you could simply substitute $t = \frac{\pi}{4}$ into the equations provided by the question.

(d) Given the function , $f(x)=\dfrac{e^{2x}+1}{2e^{x}}$. **3**

Determine if the function is odd, even or neither by showing all of your working out.

Solution:

$$f(x)=\frac{e^{2x}+1}{2e^{x}}$$

$$f(-x)=\frac{e^{2(-x)}+1}{2e^{(-x)}}$$

$$f(-x)=\frac{e^{-2x}+1}{2e^{-x}}$$

$$f(-x)=\frac{\frac{1}{e^{2x}}+1}{\frac{2}{e^{x}}}$$

$$f(-x)=\frac{\frac{1+e^{2x}}{e^{2x}}}{\frac{2}{e^{x}}}=\frac{1+e^{2x}}{e^{2x}}\times\frac{e^{x}}{2}=\frac{1+e^{2x}}{2e^{x}}=f(x)$$

Therefore the function $f(x)$ is an even function.

Marking guidelines

3 Correct solution

2 Significant working out demonstrating $f(-x)=f(x)$

1 For correctly substituting $-x$ into $f(x)$

Markers comments

Many students simply stated $f(-x)=f(x)$ without showing the working or conversely assumed $f(-x)\neq f(x)$ without showing any steps to simply the expression.

Students should be mindful that a 3 mark question typically requires many steps of working.

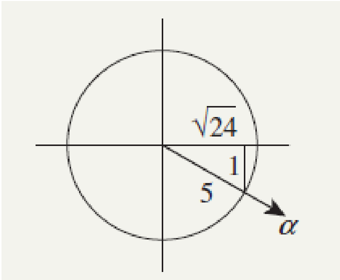
Some students have poor algebraic skills and would be encouraged to practice simplifying algebraic fractions.

Question 9 (14 marks) Use the separate writing booklet.

(a) Evaluate the expression $\tan\left(\sin^{-1}\left(-\frac{1}{5}\right)\right)$. **3**

Give your answer in the form of $a\sqrt{b}$ where a and b are rational.

Solution: Let



$\sin^{-1}(-\frac{1}{5}) = \alpha$

$\sin\alpha = -\frac{1}{5}$

$\tan\alpha = -\frac{1}{2\sqrt{6}}$

$\tan\alpha = -\frac{1}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = -\frac{\sqrt{6}}{12}$

Marking guidelines

3 Correct solution

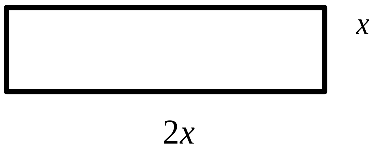
2 expression for tan α

1 expression for sin α

Markers comments

Well answered by most students

(b) The length of a rectangle remains twice as its width at all times when the rectangle is expanding. The area of the rectangle is increasing at a rate of $16\ cm^2/\sec$. Find the rate at which the perimeter of the rectangle is increasing at the instant when its width is 200 cm. **3**



$$A = 2x^2$$
$$\frac{dA}{dt} = 4x\frac{dx}{dt} = 16$$
$$\therefore x\frac{dx}{dt} = 4$$
$$when$$
$$x = 200cm$$
$$\frac{dx}{dt} = \frac{4}{200} = \frac{1}{50}\, cm/\sec$$
$$P = 6x$$
$$\frac{dP}{dt} = 6 \times 1 \times \frac{dx}{dt} = 6 \times \frac{1}{50} = \frac{3}{25}\, cm/\sec$$

Marking guidelines

3 Correct solution

2 significant steps to lead towards a correct answer

1 at least one rate of change product

Markers comments

Various strategies were used with various degrees of success. The easiest solution is $\frac{dp}{dt} = \frac{dp}{dw} \times \frac{dw}{dA} \times \frac{dA}{dt}$

To achieve at least one-mark students had to perform at least one rate of change product.

(c) The cubic equation $2x^3 - 3x + 2 = 0$ has three real roots α, β and γ.

$$2x^3 + 0 \times x^2 - 3x + 2 = 0$$
$$\alpha + \beta + \gamma = 0$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{3}{2}$$
$$\alpha\beta\gamma = -1$$

(i) **2**

$\alpha^3\beta^3\gamma^2 + \alpha^3\beta^2\gamma^3 + \alpha^2\beta^3\gamma^3$

$= \alpha^2\beta^2\gamma^2(\alpha+\beta+\gamma)$

$= (-1)^2)(-\frac{3}{2}) = -\frac{3}{2}$

Marking guidelines

2 correct final answer

1 factorising the expression

Markers comments

Well answered by most students

(ii) $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta}$ **3**

$= \frac{\alpha^2+\beta^2+\gamma^2}{\alpha\beta\gamma}$

$= \frac{(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)}{\alpha\beta\gamma}$

$= \frac{0^2 - 2\times -\frac{3}{2}}{-1} = -3$

Marking guidelines

3 Correct final answer

2 correct factorisation (and simplifying)

1 correct common denominator

Markers comments

Many students achieved full marks. Student who did not, struggled to simplify the fractions correctly and convert the numerator. There were numerous unforced errors due to poor setting out or rushed working out. Students who accidentally gave the correct answer did not receive any marks.

(d)

Using the t formula , solve $\sin x = 3\cos x + 2$, for $0^\circ \le x \le 360^\circ$ to the nearest degree. **3**

$\frac{2t}{1+t^2} = 3\frac{(1-t^2)}{1+t^2} + 2$

$2t = 3(1-t^2) + 2(1+t^2)$

$t^2 + 2t - 5 = 0$

$t = \frac{-2 \pm \sqrt{24}}{2} = -1 \pm \sqrt{6}$

$\tan\frac{x}{2} = -1+\sqrt{6}$

$x = 2\times\tan^{-1}(-1+\sqrt{6}) \approx 111^\circ$

$x = 2\times(180^\circ - \tan^{-1}(1+\sqrt{6})) \approx 212^\circ$

$\because 0^\circ \le x \le 360^\circ$

Marking guidelines

3 Correct final answers (both angles)

2 One correct answer

1 Correct quadratic expression in terms of *t*

Markers comments

Most students substituted correctly and rearranged to obtain the quadratic formula. But many either did not continue or could not obtain the correct answer. Students need to make sure they do not use rounded off numbers in their calculations.

Question 10 (13 marks) Use the separate writing booklet.

(a) $P(x)=x^4+7x^3+9x^2-27x+c$ has a zero of multiplicity 3. **3**

Find the value of c .

$P'(x)=4x^3+21x^2+18x-27$

$P''(x)=12x^2+42x+18$

$P''(x)=0$

$2x^2+7x+3=0$

$x=-\frac{1}{2}$

Or

$x=-3$

$P'(-\frac{1}{2})=-\frac{125}{4}\neq 0$

$P'(-3)=0$

Therefore $x=-3$ is a zero of $P(x)$.

$P(-3)=(-3)^4+7\times(-3)^3+9(-3)^2-27\times(-3)+c=0$

$c=-54$

Marking guidelines

3 Correct solution

2 Significant working out; checking if $P'(-3)=0 \ or \ P'\left(-\frac{1}{2}\right)=0$

1 For finding $x=\frac{-1}{2} or -3$

Markers comments

Many students didn't use the multiple roots theorem.

Some students used the multiple roots theorem but didn't apply it correctly by failing to check if

$P'(-3)=0 \ or \ P'\left(-\frac{1}{2}\right)=0$

(b) If α and β are acute angles such that $\tan\alpha=\frac{m}{m+1}$ and $\tan\beta=\frac{1}{2m+1}$. **3**

Prove that $\alpha+\beta=\frac{\pi}{4}$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}\times\frac{1}{2m+1}}$$

$$\tan(\alpha+\beta)=\frac{2m^2+2m+1}{2m^2+2m+1}=1$$

$$\tan(\alpha+\beta)=1=\tan\frac{\pi}{4}$$

$$\therefore \alpha+\beta=\frac{\pi}{4}$$

Marking guidelines

3 Correct solution

2 Significant working out; Simplifying the algebraic fraction to $\frac{2m^2+2m+1}{2m^2+2m+1}$

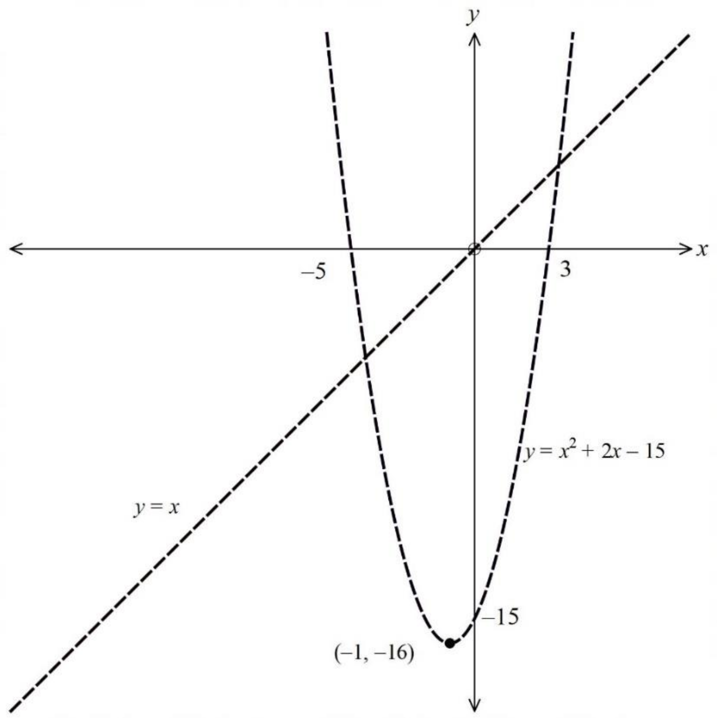
1 Correct substitution into $\tan\left(\alpha+\beta\right)$

Markers comments

Many students didn't know the correct formula for $\tan\left(\alpha+\beta\right)$

(c) A function is defined as $f(x) = x^2 + 2x - 15$.

The graphs of $y = f(x)$ and of the line $y = x$ are shown on the diagram below.



(i) Determine what restriction including its y - intercept would need to be put on the domain of $f(x)$ if it is to have an inverse function $f^{-1}(x)$. **1**

$x \geq -1$

Marking guidelines

1 Correct solution

Markers comments Well done

(ii) Determine the equation of the inverse function $f^{-1}(x)$. **2**

$$x = y^2 + 2y - 15$$
$$x + 15 = y^2 + 2y$$
$$x + 15 + 1 = y^2 + 2y + 1$$
$$(y+1)^2 = x + 16$$
$$y = -1 \pm \sqrt{x+16}$$
$$y = -1 + \sqrt{x+16} (y \geq -1)$$
$$f^{-1}(x) = -1 + \sqrt{x+16}$$

Marking guidelines

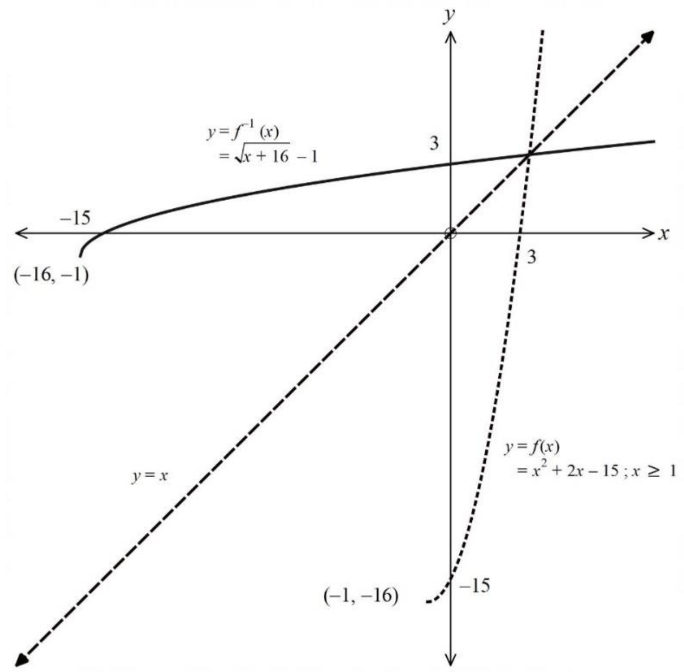
2 Correct solution

1 Swapping x and y.

Markers comments

Many students did not make *y* the subject. Many students didn't include plus or minus when square rooting both sides. Some students don't understand how to determine if the positive or negative square root is required.

(iii) Draw a sketch showing the graph of $y = f^{-1}(x)$. **1**



Marking guidelines

1 Correct graph

Markers comments Well done but a few students didn't mark the intercepts with the axes

(d) Given $\sec x + \tan x = k$, Find the value of $\cos x$ in terms of k. **3**

$$\sec x + \tan x = k \ldots\ldots\ldots\ldots (i)$$
$$\sec^2 x - \tan^2 x = 1$$
$$(\sec x + \tan x)(\sec x - \tan x) = 1$$
$$k \times (\sec x - \tan x) = 1$$
$$\sec x - \tan x = \frac{1}{k} \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$
$$(i) + (ii)$$
$$2\sec x = k + \frac{1}{k} = \frac{k^2+1}{k}$$
$$\frac{2}{\cos x} = \frac{k^2+1}{k}$$
$$\therefore \cos x = \frac{2k}{k^2+1}$$

Marking guidelines

3 Correct solution

2 Significant working out; Adding equations 1 and 2 .

1 Making correct use of trigonometric identities

Markers comments

Not well done.

Alternate solution for (d)

$$\sec x + \tan x = k$$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x} = k....(*)$$

$$1 + \sin x = k \cos x$$

$$\sqrt{1-\cos^2 x} = k \cos x - 1$$

$$1 - \cos^2 x = k^2 \cos^2 x - 2k \cos x + 1$$

$$0 = k^2 \cos^2 x + \cos^2 x - 2k \cos x$$

$$0 = \left(k^2 + 1\right)\cos^2 x - 2k \cos x$$

$$0 = \cos x\left(\left(k^2 + 1\right)\cos x - 2k\right)$$

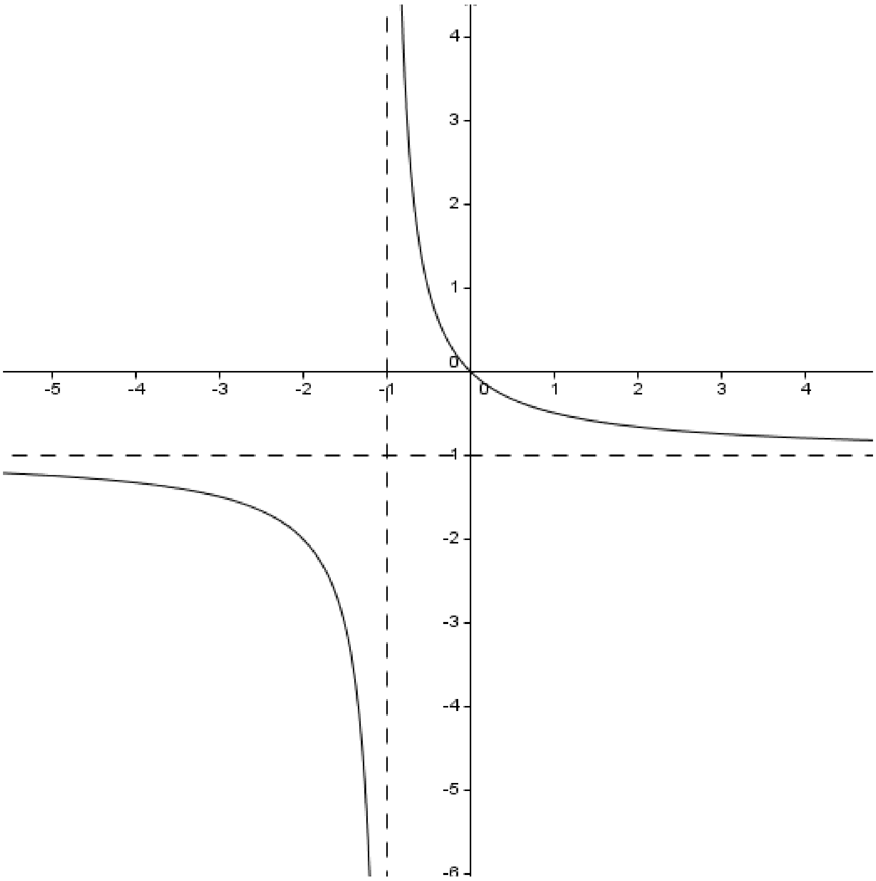
$$\cos x = 0 \text{ or } \frac{2k}{k^2+1}$$

$\therefore \cos x$ written in terms of k is $\cos x = \dfrac{2k}{k^2+1}$

Also, $\cos x = 0$ may be excluded as it makes (*) undefined.

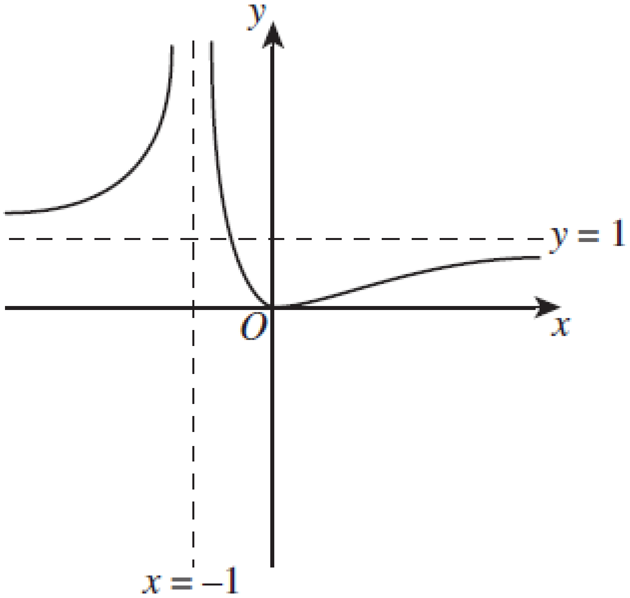
Question 11 (13 marks) Use the separate writing booklet.

The diagram below is a sketch of the graph of the function $f(x) = -\dfrac{x}{x+1}$



(a) Sketch the graph of $y = (f(x))^2$, showing all the asymptotes and intercepts.
y

(i)



Marking guidelines

2 Correct solution

1 for partially correct

Markers comments

well done

Sketch the graph of $y = x + f(x)$,showing all asymptotes and intercepts.

(ii)

Marking guidelines

1 Correct solution

Markers comments

Not well done

(b) In a group of 1000 computers linked to each other via the internet, the number N infected with a virus at time t years is given by

$$N = \frac{1000}{1 + ce^{-1000t}}$$

Where c is a constant.

(i) Show that , eventually , all the computers will be infected with the virus. **1**

When

$t \to \infty$

$e^{-1000t} \to 0$

$N \to 1000$

Marking guidelines

1 Correct solution

Markers comments Well done but a few students didn't realise to take $t \to \infty$

2

(ii) At the beginning ,only one computer was infected with the virus.
After how many days will the 50% of the computers be infected?

$$t = 0, N = 1$$
$$\therefore c = 999$$
$$500 = \frac{1000}{1 + ce^{-1000t}}$$
$$ce^{-1000t} = 1$$
$$t = -\frac{1}{1000}\ln\left(\frac{1}{c}\right) = -\frac{1}{1000}\ln\left(\frac{1}{999}\right) \approx 6.90675 \times 10^{-3}\, years \approx 2.52 days$$

Marking guidelines

2 Correct solution

1 for some progress

Markers comments
Well done but a few students didn't give the time in days as asked in the question.

2

(ii) Show that $\frac{dN}{dt} = N(1000 - N)$

$$\frac{dN}{dt} = 1000 \times (-1) \times (1 + ce^{-1000t})^{-2} \times ce^{-1000t} \times (-1000)$$
$$N(1000 - N) = \frac{1000}{1 + ce^{-1000t}}\left(1000 - \frac{1000}{1 + ce^{-1000t}}\right) = \frac{\ldots}{1 + \ldots} \ldots \frac{\ldots e^{-1000t}}{(1 + ce^{-1000t})^2}$$
$$\therefore \frac{dN}{dt} = N(1000 - N)$$

Marking guidelines

2 Correct solution

1 for some progress

Markers comments
Well done but a few students didn't differentiate properly

(c) The table shows selected values of a one-to-one differential function and its derivative $g'(x)$.

x	-1	0
$g(x)$	-5	-1
$g'(x)$	3	$\frac{1}{2}$

Let $f(x)$ be a function such that $f(x) = g^{-1}(x)$.

Find the value of $f'(-1)$.

Justify your answer with mathematical reasoning.

Solution 1

From the table, $f(x) = g^{-1}(x)$ and so $f(-1) = g^{-1}(-1) = 0$.

$$f'(-1) = \frac{1}{g'(f(-1))}$$

$$= \frac{1}{g'(0)}$$

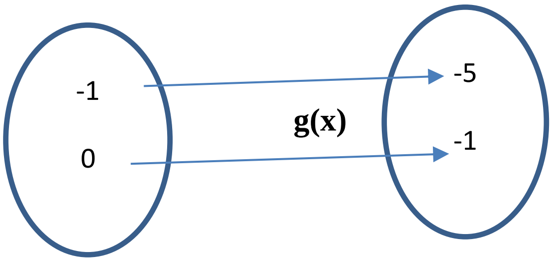
$$= \frac{1}{\frac{1}{2}}$$

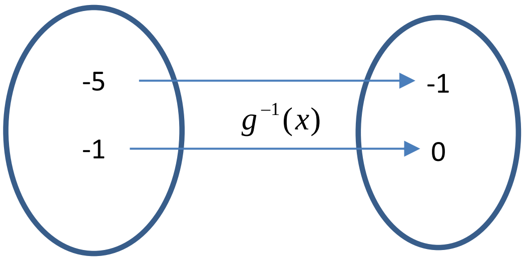
$$= 2$$

Or

Using the graphs to the concept of the graph of an inverse function of a given function is the reflection of the original graph in $y = x$

Or





$$f(x) = g^{-1}(x)$$
$$g(f(x)) = g(g^{-1}(x))$$
$$g(f(x)) = x$$

Taking Derivatives on both sides

$$g'(f(x)) \times f'(x) = 1$$
$$f'(x) = \frac{1}{g'(f(x))}$$
$$f'(-1) = \frac{1}{g'(f(-1))}$$
$$f(-1) = g^{-1}(-1) = 0$$
$\therefore$
$$f'(-1) = \frac{1}{g'(0)} = \frac{1}{\frac{1}{2}} = 2$$

Marking guidelines

3 Correct solution

2 significant progress towards the solution

1 finding correct value of $f(-1)$

Markers comments

Very poorly done

End of Paper