Caringbah High School

Year 11 2025

Mathematics Extension 1

Preliminary Course

Assessment Task 3

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen
- Board-approved calculators may be used
- No liquid paper or tape may be used
- In Questions 6-10, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 60

Section I **5 marks**

Attempt Questions 1-5

Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **55 marks**

Attempt Questions 6-10

Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Student Number: ___________________ **Class:** ____________

Marker's Use Only							
Section I	**Section II**					**Total**	
Q1-5	**Q6**	**Q7**	**Q8**	**Q9**	**Q10**		
/5	/10	/12	/12	/11	/10	/60	%

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Section I

5 marks
Attempt Questions 1-5
Allow about 8 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–5

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1. Find the remainder when $P(x) = 4x^3 - x^2 + 7$ is divided by $x - 1$.

(A) 2

(B) 4

(C) 10

(D) 12

2. What is the domain and range of $y = 2\sin^{-1}\left(\frac{x}{3}\right)$?

(A) The domain is $-1 \leq x \leq 3$ and the range is $-\pi \leq y \leq \pi$

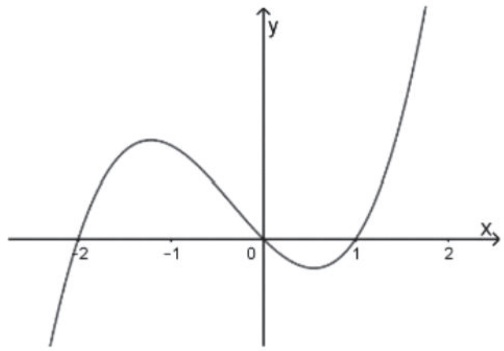
(B) The domain is $-\pi \leq x \leq \pi$ and the range is $-3 \leq y \leq 3$

(C) The domain is $-3 \leq x \leq 3$ and the range is $-3 \leq y \leq 3$

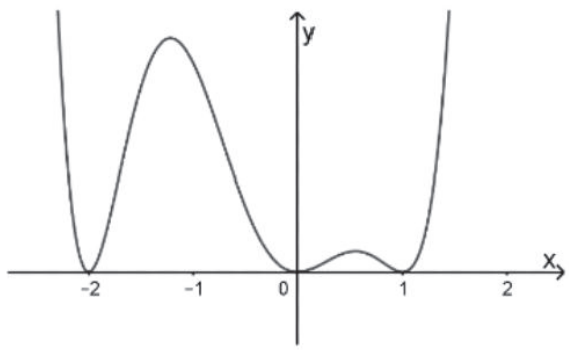
(D) The domain is $-3 \leq x \leq 3$ and the range is $-\pi \leq y \leq \pi$

Section I continues next page

3 The graph of the function $y = f(x)$ is shown below



A second graph is obtained from the function above is shown below



Which equation could best represent the second graph?

(A) $y = \sqrt{f(x)}$

(B) $y = |f(x)|$

(C) $y = f(|x|)$

(D) $y = (f(x))^2$

Section I continues next page

4. Which of following is an equivalent expression for $\dfrac{1}{1-\tan x}-\dfrac{1}{1+\tan x}$?

(A) $\dfrac{2\tan x}{\sec^2 x}$

(B) $\tan(2x)$

(C) $\dfrac{\tan(2x)}{\tan x}$

(D) $\tan x \tan(2x)$

5. What is the smallest size of a set of whole numbers that will guarantee that at least three of the numbers have the same remainder after division by 7?

(A) 15

(B) 3

(C) 14

(D) 22

End of Section I

Section II

55 marks
Attempt Questions 6–10
Allow about 82 minutes for this section

Answer each question in a SEPARATE writing booklet.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) Use a SEPARATE writing booklet.

(a) Solve **2**

$$|2x + 1| \geq 3$$

(b) Consider the polynomial $P(x) = x^3 + 5x^2 + 3x - 9$.

(i) Show that $P(x)$ has a double root at $x = -3$. **2**

(ii) Hence, or otherwise, factorise $P(x)$ completely. **1**

(c) How many possible distinct arrangements can be made by using the letters of the word POSSIBILITY?

(i) with no restrictions? **1**

(ii) If both letters **S** are together? **1**

(iii) If the letter **S** occupies both first and last place? **1**

(d) Find the Cartesian equation of the curve with parametric equations: **2**

$$x = 20t \text{ and } y = 40t - 5t^2$$

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) Solve **3**

$$\frac{2}{x-1} \geq x$$

(b) Show that there exists only one value of a for which the polynomial **2**

$P(x) = x^4 + 2x^3 - x^2 - 8x - a$ is divisible by $Q(x) = x^2 - 4$.

(c) Find the number of ways that a group of four students can be chosen from six juniors and three seniors:

(i) without restrictions. **1**

(ii) so that there are unequal numbers of juniors and seniors. **2**

(d) Consider the function $f(x) = (x-2)^2,\ 0 \leq x \leq 2$. **2**

Find the inverse function $f^{-1}(x)$ and state its domain and range.

(e) Using $t = \tan\left(\frac{x}{2}\right)$, or otherwise, prove that **2**

$$\frac{2}{1 + \cos x} = \sec^2\left(\frac{x}{2}\right)$$

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) The polynomial $P(x) = x^3 - 2x^2 + kx + 24$, for some real value k, has roots α, β, γ.

(i) Find the values of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$. **1**

(ii) It is known that two of the roots are equal in magnitude but opposite in sign. Find the third root, and hence, find the value of k. **2**

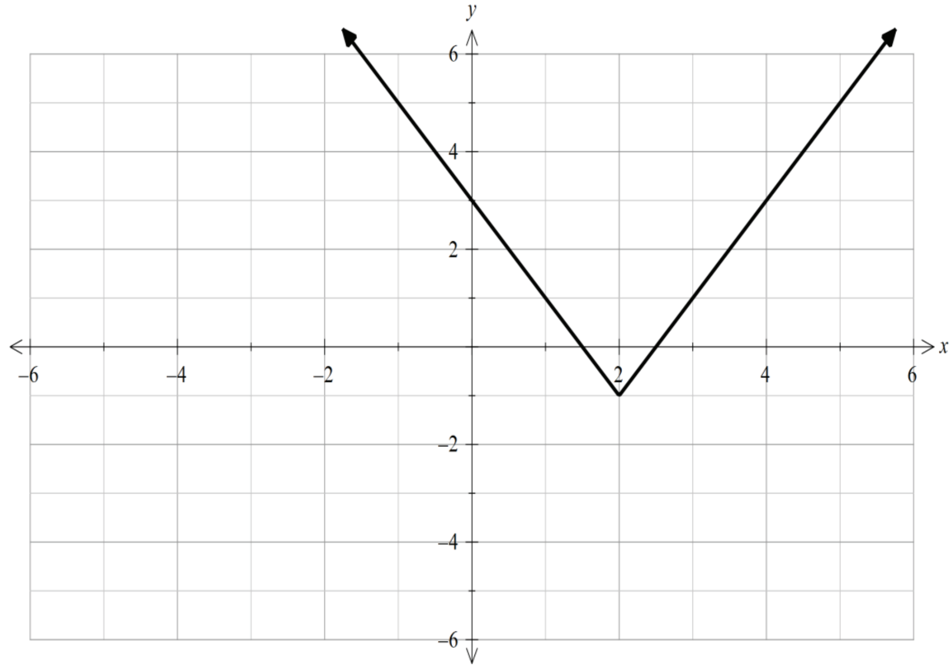
(b) Solve, correct to the nearest degree. **2**

$$2\cos(2\theta) = 4\cos\theta - 3, \quad 0° \leq \theta \leq 360°$$

(c) A set of five distinct parallel lines intersects a different set of seven distinct parallel lines. How many possible parallelograms can be made from these lines? **2**

Question 8 continues next page

(d) The graph $y = f(x)$ is shown below.



(i) On Question 8 template, sketch the graph $y = |f(x)|$. **2**

(ii) Hence or otherwise, solve the equation $|f(x)| + 3 = 4$ **1**

(iii) On Question 8 template, sketch separately the graph $y^2 = f(x)$, showing all intercepts and intersection points with $y = f(x)$. **2**

End of Question 8

Question 9 (11 marks) Use a SEPARATE writing booklet.

(a) A curve is defined by the parametric equations: **2**

$$x = \operatorname{cosec}(\theta) + 1$$
$$y = 2\cot(\theta)$$

Find, for $0 \le \theta \le 2\pi$, the Cartesian equation of the curve.

(b) Prove that **3**

$$\cos\left(\sin^{-1}\left(\frac{1}{4}\right) - \tan^{-1}\left(-\frac{1}{2}\right)\right) = \frac{10\sqrt{3} - \sqrt{5}}{20}$$

(c) The digits 1, 2, 3, 4, 5, 6 are spaced evenly around a circle. **2**
Each digit has another digit which is directly opposite it on the circle.
If the digits are placed around the circle at random, what is the probability that exactly one pair of opposite digits add to 7?

(d) The polynomial $P(x)$ is of degree 3, and $P(x) - 1$ is divisible by $(x-1)^2$.

(i) Find the value of $P(1)$. **1**

(ii) Show that $P'(x)$ is divisible by $(x-1)$. **1**

(iii) Given also that $P'(x)$ is divisible by $(x+1)$ and $P(-1) = -1$. **2**
Find $P(x)$.

End of Question 9

Question 10 (10 marks) Use a SEPARATE writing booklet.

(a) The quadratic equation $ax^2 + bx + c = 0$ has roots $x_1 = \tan\alpha$ and $x_2 = \tan\beta$.

(i) Show that $\tan(\alpha + \beta) = \dfrac{b}{c - a}$. **2**

(ii) Hence, or otherwise, show that $\tan^2(\alpha - \beta) = \dfrac{b^2 - 4ac}{(a + c)^2}$. **2**

(b) The letters of the word IMPRESSION are rearranged such that the letters of the word PRESS appear in order but not necessarily together. **2**

One such arrangement is O**P**N**RE**I**S**M**S**I.

How many such arrangements are there?

(c) Solve **4**

$$\cos^2 x + \cos^2(2x) + \cos^2(3x) = 1,\ 0 \le x \le 2\pi$$

End of Examination 😊

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Section I Multiple Choice Answer Sheet

Student Name : ______________________________ Class : __________

If you detach this sheet, please make sure your name is written above.

Completely fill the response oval representing the most correct answer.

Sample:

2 + 4 = ? (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ☒ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ↘

A ● B ☒ C ○ D ○

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1 A ○ B ○ C ○ D ○

2 A ○ B ○ C ○ D ○

3 A ○ B ○ C ○ D ○

4 A ○ B ○ C ○ D ○

5 A ○ B ○ C ○ D ○

2025 Yr 11 Ext AT3 (V2) - Solutions

Section 1

Q1	Q2	Q3	Q4	Q5
B	C	A	B	D

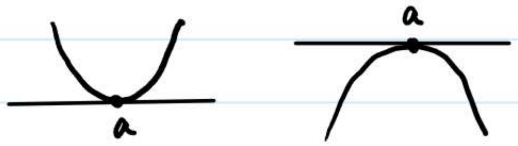
1. $P(x) = x^3 - 12x + 10$

$P(3): 3^3 - 12(3) + 10 = 1$ (B)

2. $2\sin 3x \sin 5x = \cos(5x - 3x) - \cos(5x + 3x)$

$= \cos 2x - \cos 8x$ (C)

3.



f(x) sign remains

f'(x) sign changes (A)

4. $P(x) = x^3 - 8x^2 + ax - b$

$P'(x) = 3x^2 - 16x + a$

$P(2) = P'(2) = 0$

$P'(2) = 3(2)^2 - 16(2) + a = 0$

$a - 20 = 0$

$a = 20$

$P(2) = 2^3 - 8(2)^2 + 20(2) - b = 0$

$16 - b = 0$

$b = 16$ (B)

1. What is the remainder when the polynomial $P(x) = x^3 - 12x + 10$ is divided by $(x - 3)$?

(A) –53

(B) 1

(C) –10

(D) 73

2. Which of the following is equivalent to $2\sin 3x \sin 5x$?

(A) $-\cos 2x - \cos 8x$

(B) $\cos 8x - \cos 2x$

(C) $\cos 2x - \cos 8x$

(D) $\cos 2x + \cos 8x$

3 The function $f(x)$ has a root of even multiplicity at $x = a$.
Which of the following best describes the sign change of $f(x)$ and the sign change of $f'(x)$ on either side of $x = a$?

(A) The sign of $f(x)$ does not change, but the sign of $f'(x)$ changes.

(B) The sign of $f(x)$ changes, but the sign of $f'(x)$ does not change.

(C) The signs of both $f(x)$ and $f'(x)$ do not change.

(D) The signs of both $f(x)$ and $f'(x)$ change.

4. The polynomial $P(x) = x^3 - 8x^2 + ax - b$ has a double root at $x = 2$.
Which of the following are the values of a and b?

(A) $a = -44, b = 48$

(B) $a = 20, b = 16$

(C) $a = -44, b = -48$

(D) $a = 20, b = -16$

5. $f(x) = 2\cos x, \quad 0 \le x \le \pi$

$g(x) = 4x^2 - 6$

$h(x) = f^{-1}(x) = \cos^{-1}\left(\frac{x}{2}\right), \quad -2 \le x \le 2$

$h(g(x)) = h(4x^2 - 6)$

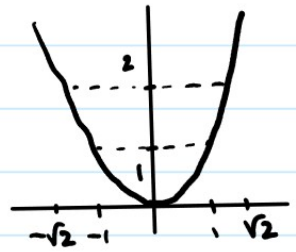
$= \cos^{-1}\left(\frac{4x^2-6}{2}\right)$

D: $-1 \le \frac{4x^2-6}{2} \le 1$

$-2 \le 4x^2 - 6 \le 2$

$4 \le 4x^2 \le 8$

$1 \le x^2 \le 2$



$-\sqrt{2} < x < -1, \quad 1 < x < \sqrt{2}$

(D)

5. Two functions, $f(x)$ and $g(x)$, are defined below. The inverse of $f(x)$ is $h(x)$.

$$f(x) = 2\cos x \text{ for } 0 \le x \le \pi$$

$$g(x) = 4x^2 - 6 \text{ for all real } x$$

What is the domain of $h(g(x))$?

(A) $-2 \le x \le 2$

(B) $0 \le x \le \pi$

(C) All real x

(D) $-\sqrt{2} < x < -1$ or $1 < x < \sqrt{2}$

Section 2

Question 6

(a) $|3-2x| \leq 7$

$-7 \leq 3-2x \leq 7$ ✓

$-10 \leq -2x \leq 4$

$5 \geq x \geq -2$

$\therefore -2 \leq x \leq 5$ ✓

(b) $y = 4\cos^{-1}\left(\frac{2x}{3}\right)$

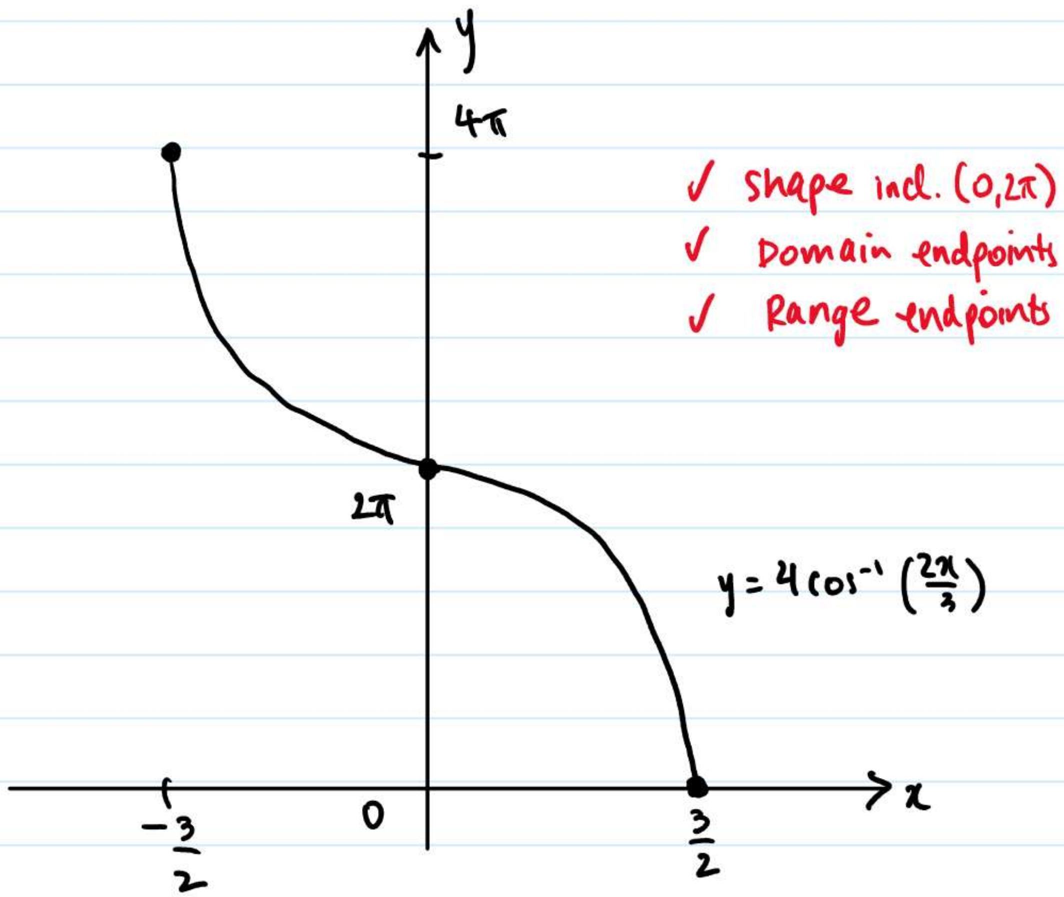
D: $-1 \leq \frac{2x}{3} \leq 1$

$-\frac{3}{2} \leq x \leq \frac{3}{2}$

R: $0 \leq \cos^{-1} u \leq \pi$

$0 \leq 4\cos^{-1} u \leq 4\pi$

$0 \leq y \leq 4\pi$



Question 6 (11 marks) Use a SEPARATE writing booklet.

(a) Solve $|3-2x| \leq 7$ **2**

Generally answered well.

(b) Draw a neat sketch of the function below, labelling key features. **3**

$$y = 4\cos^{-1}\left(\frac{2x}{3}\right)$$

Answered poorly. It is recommended that students review how to adjust the domain and range of inverse trig functions. 1 mark was allocated for the correct domain, 1 for the correct range, and a third mark was awarded for a correct shape and y intercept being labelled.

(c) HANNAH

(i) $\dfrac{6!}{2!\times 2!\times 2!} = 90$ ✓

(ii) (HH) _ _ _ _

$\dfrac{5!}{2!\times 2!\times 2!} = 15$ ✓

$P(HH) = \dfrac{15}{90} = \dfrac{1}{6}$ ✓

(d) Let $t = \tan\frac{\theta}{2}$, then $\tan\theta = \dfrac{2t}{1-t^2}$

Sub $\theta = 315°$, $t = \tan 157.5°$

$\tan 315° = \dfrac{2t}{1-t^2}$

$-1 = \dfrac{2t}{1-t^2}$ ✓

$-1 + t^2 = 2t$

$t^2 - 2t - 1 = 0$

$t = \dfrac{2 \pm \sqrt{8}}{2}$

$= \dfrac{2 \pm 2\sqrt{2}}{2}$

$= 1 \pm \sqrt{2}$ ✓

But $\tan 157.5° < 0$ (2nd quadrant)

$\therefore \tan 157.5° = 1 - \sqrt{2}$ ✓

Must explain why $\tan 157.5° \neq 1 + \sqrt{2}$

(c) The letters in HANNAH are to be arranged in a line.

(i) In how many different ways can the letters be arranged? **1**

(ii) Find the probability that the two Hs will be next to each other. **2**

(i) Mixed results. A handful forgot to divide by the repeated elements.

(ii) Mixed results. The same issue arose with students forgetting to divide by the repeated elements.

(d) Using t-results, find the exact value of $\tan 157.5°$ **3**

Answered poorly. To achieve the first mark students had to recognise that $-1 = \frac{2t}{1-t^2}$. Students then needed to solve the quadratic and omit the positive answer by explaining that $\tan(157.5°) < 0$.

Question 7

(a) $\frac{2}{x+1} \le x \quad [x \neq -1]$

$\frac{2}{x+1} - x \le 0$

$\frac{2 - x(x+1)}{x+1} \le 0$ ✓

$(x+1)^2 \times \frac{2 - x^2 - x}{x+1} \le 0 \times (x+1)^2$

$-(x+1)(x^2+x-2) \le 0$

$-(x+1)(x+2)(x-1) \le 0$ ✓

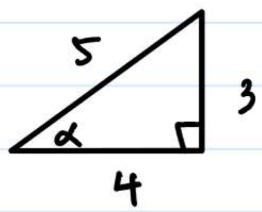
$\therefore -2 \le x < -1, \; x \ge 1$ ✓

↑ No equality

(b) $\tan\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)$

$\alpha = \sin^{-1}\left(\frac{3}{5}\right)$

$\sin\alpha = \frac{3}{5}$



✓

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

$= \frac{2(3/4)}{1-(3/4)^2}$

$= \frac{24}{7}$ ✓

Question 7 (11 marks) Use a SEPARATE writing booklet.

(a) Solve **3**

$$\frac{2}{x+1} \le x$$

Generally well-done, some students forgot to include the case of x is different to -1, so be careful to always include discontinuities.
Some students had poor working out, you need to convince the marker in your working you know what you are doing. If you are doing critical point method, show the testing of the regions etc.

(b) Showing all working, find the exact value of **2**

$$\tan\left(2\sin^{-1}\left(\frac{3}{5}\right)\right)$$

Some students used calculator and didn't show working to get $\frac{24}{7}$, working needs to be shown for 2/2 marks.

(c) $P(x) = x^3 + ax + b$

$$\begin{cases} P(2) = 0 \\ P(-1) = -15 \end{cases}$$

$P(2) = 2^3 + 2a + b = 0$

$2a + b = -8$ — ①

$P(-1) = (-1)^3 + a(-1) + b = -15$

$-a + b = -14$ — ②

① - ②: $3a = 6$

$\therefore a = 2,\ b = -12$

(d) 3 couples, 2 singles

(i) (CC) (CC) (CC) S S

$5! \times 2! \times 2! \times 2! = 960$

(ii) Total $= 8!$

Separate $=$ Total $-$ Together

$= 8! - 960$

$= 39360$

Probability $= \dfrac{39360}{8!}$

$= \dfrac{41}{42}$

(c) The polynomial $P(x)$ is given by $P(x) = x^3 + ax + b$ for real numbers a and b. The equation $P(x) = 0$ has a root at $x = 2$. When $P(x)$ is divided by $(x + 1)$, the remainder is -15. Find the values of a and b. **3**

Mostly well done, some arithmetic mistakes, a lot of students added $-15 + 1$ to -16 rather than -14, don't rush the working.

(d) A party of eight people are going to the movies. There are three couples and two single individuals.

(i) In how many ways can this group sit in a row of eight seats at the movies, such that each member of a couple is sitting beside their partner? **1**

(ii) If the party were placed at random in the row of seats, what is the probability that the members of at least one couple are not sitting together? **2**

(i) Many got this part correctly. Some included an extra 2!, be careful.

(ii) Some students didn't realise it is a probability question and lost a mark.

Question 8

a) $S(x) = 3x^3 - 4x^2 + 6x - 1$

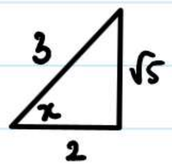
(i) $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{6}{3} = 2$ ✓

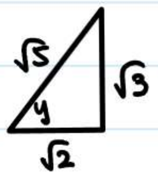
(ii) $\alpha + \beta + \gamma = --\frac{4}{3} = \frac{4}{3}$

$\alpha\beta\gamma = --\frac{1}{3} = \frac{1}{3}$ ✓

$\therefore \alpha + \beta + \gamma = 4\alpha\beta\gamma$

(b) $\sin x = \frac{\sqrt{5}}{3}$ $\sin y = \frac{\sqrt{3}}{\sqrt{5}}$





✓

$\sin(x+y) = \sin x \cos y + \sin y \cos x$

$= \frac{\sqrt{5}}{3} \times \frac{\sqrt{2}}{\sqrt{5}} + \frac{\sqrt{3}}{\sqrt{5}} \times \frac{2}{3}$

$= \frac{\sqrt{10} + 2\sqrt{3}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{50} + 2\sqrt{15}}{15}$

$= \frac{5\sqrt{2} + 2\sqrt{15}}{15}$ ✓ Don't need to rationalise denom.

Question 8 (11 marks) Use a SEPARATE writing booklet.

(a) A polynomial $S(x) = 3x^3 - 4x^2 + 6x - 1$ has roots α, β and γ.

(i) Find the value of $\alpha\beta + \beta\gamma + \alpha\gamma$ **1**

(ii) Show that the sum of the roots is four times the product of the roots. **1**

Very well done for both parts (i) and (ii)

(b) Given that for $0 \le x \le \frac{\pi}{2}$ and $0 \le y \le \frac{\pi}{2}$ **2**

$$\sin x = \frac{\sqrt{5}}{3}$$

$$\sin y = \frac{\sqrt{3}}{\sqrt{5}}$$

Find the exact value of $\sin(x+y)$.

Care should be taken to show where the values for $\cos x$ and $\cos y$ are derived. Also there were many careless errors in the process of simplifying surds.

(c) $\begin{cases} x = 2 + 3\cos t & — ① \\ y = -1 + 3\sin t & — ② \end{cases}$

From ①, $x - 2 = 3\cos t$

$\cos t = \frac{x-2}{3}$ — ③

From ②, $y + 1 = 3\sin t$

$\sin t = \frac{y+1}{3}$ — ④ ✓

$③^2 + ④^2$: $\cos^2 t + \sin^2 t = \left(\frac{x-2}{3}\right)^2 + \left(\frac{y+1}{3}\right)^2$

$1 = \frac{(x-2)^2}{9} + \frac{(y+1)^2}{9}$

$\therefore (x-2)^2 + (y+1)^2 = 9$ ✓

$C(2, -1)$

$r = 3$ units ✓

(d) (i) $f(x)$: $y = 1 + e^x$

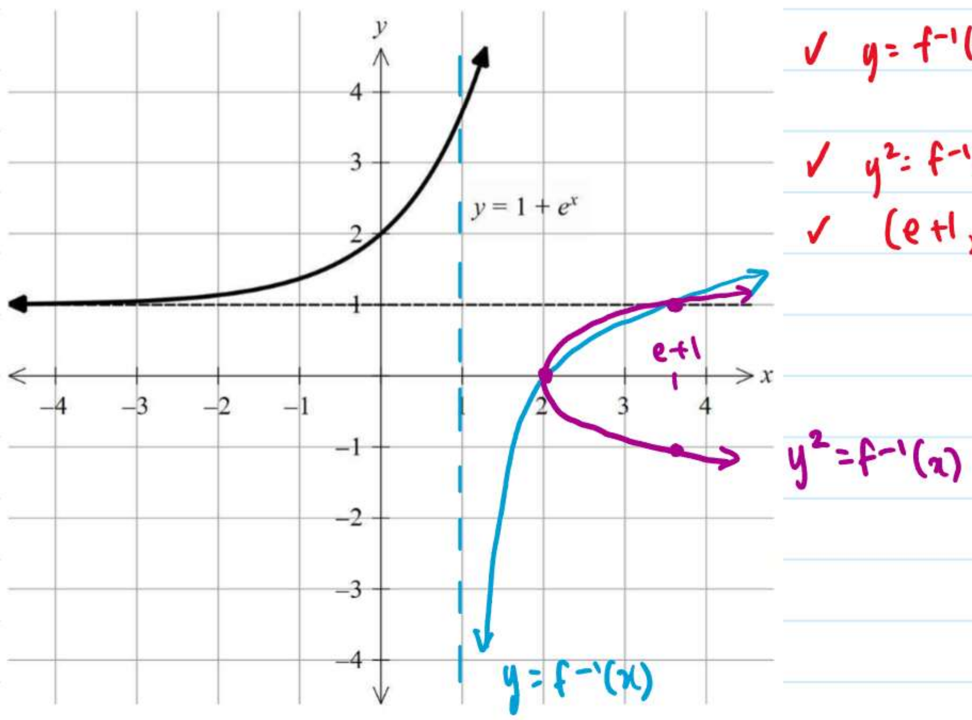
$f^{-1}(x)$: $x = 1 + e^y$

$x - 1 = e^y$

$y = \ln(x-1)$ ✓ (show)

$\therefore f^{-1}(x) = \ln(x-1)$

(ii)



✓ $y = f^{-1}(x)$

✓ $y^2 = f^{-1}(x)$ shape

✓ $(e+1, \pm 1)$

(c) Derive the Cartesian equation of the curve represented by the parametric equations below and show that it represents a circle. State the centre and radius of the circle. **3**

$$x = 2 + 3\cos t$$
$$y = -1 + 3\sin t$$

Most students were able to rearrange to make cos or sin the subject, many didn't use the $\sin^2 x + \cos^2 x = 1$ identity and found the question quite difficult.

(d) Below is the graph of $f(x) = 1 + e^x$.

(i) Show that the inverse function $f^{-1}(x) = \ln(x-1)$ **1**

(ii) **On the Question 8 template** sketch the graphs of $y = f^{-1}(x)$ and $y^2 = f^{-1}(x)$ on the same set of axes. Label all key features and intercepts with the coordinate axes. **3**

(i) Two acceptable methods, either swap the x and y, then rearrange to make y the subject, or show at least part of this:

$f(f^{-1}(x)) = x = f^{-1}(f(x))$

(ii) Most students were able to graph $y = f(x)$ well.

The graph of $y^2 = f(x)$ was more challenging, I mostly marked based on the x-intecept and the shape of the graph.
Very few students looked to draw the point where the two graphs intersect (or where either intersect the line $y = 1$). This needed to be evaluated in exact form for the final mark.

Question 9

(a) RTP: $\cot\theta - \tan\theta = 2\cot 2\theta$

LHS $= \cot\theta - \tan\theta$

$= \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$ ✓

$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta}$

$= 2\cot 2\theta$ ✓ (Prove)

$=$ RHS

(b) $4x^3 - 27x + k = 0$

Let roots be α, α, β

① $\alpha + \alpha + \beta = 0$

$2\alpha + \beta = 0$

$\beta = -2\alpha$

② $\alpha\alpha + \alpha\beta + \alpha\beta = -27/4$ ✓

$\alpha^2 + 2\alpha\beta = -27/4$

$\alpha^2 - 4\alpha^2 = -27/4$

$-3\alpha^2 = -27/4$

$\alpha^2 = 9/4$

$\alpha = \pm\frac{3}{2}$, $\beta = \mp 3$ ✓

③ $\alpha\alpha\beta = -k/4$

$\alpha^2\beta = -k/4$

$k = -4\alpha^2\beta$

$= -4 \times \frac{9}{4} \times \pm 3$

$\therefore k = \pm 27$ ✓

Question 9 (11 marks) Use a SEPARATE writing booklet.

(a) Prove that $\cot\theta - \tan\theta = 2\cot 2\theta$ **2**

Mixed results. Many students skipping steps, which is not advised for any prove/show questions. Some students clearly need to familiarise themselves with the double angle formulae for sine and cosine. Many students unable to simplify $2\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) = \frac{1-\tan^2\theta}{\tan\theta}$ straight away, instead expanding $2\left(\frac{1-\tan^2\theta}{2\tan\theta}\right) = \frac{2-2\tan^2\theta}{2\tan\theta}$ then simplifying – very inefficient!!

(b) Find the possible values of k such that $4x^3 - 27x + k = 0$ has two equal roots. **3**

Mixed results. Students who used $f'(x) = 0$ for a double root were generally able to answer correctly, though most students left $\alpha = \pm\sqrt{\frac{27}{12}}$ and not simplifying to $\alpha = \pm\frac{3}{2}$. Students who did not consider $\pm$ cases only found one answer for k, so did not gain full marks. Students who used relations of roots often made errors with the sign of $\frac{c}{a}$ and $-\frac{d}{a}$. Some students tried to use the discriminant, which does not work with cubic equations!

(c) 6 male, 4 female

(i) ${}^6C_3 \times {}^4C_3 = 80$ ✓

(ii) Case 1: Male on

$1 \times {}^8C_5 = 56$

Case 2: Female on

$1 \times {}^8C_5 = 56$ } ✓

Case 3: Neither on

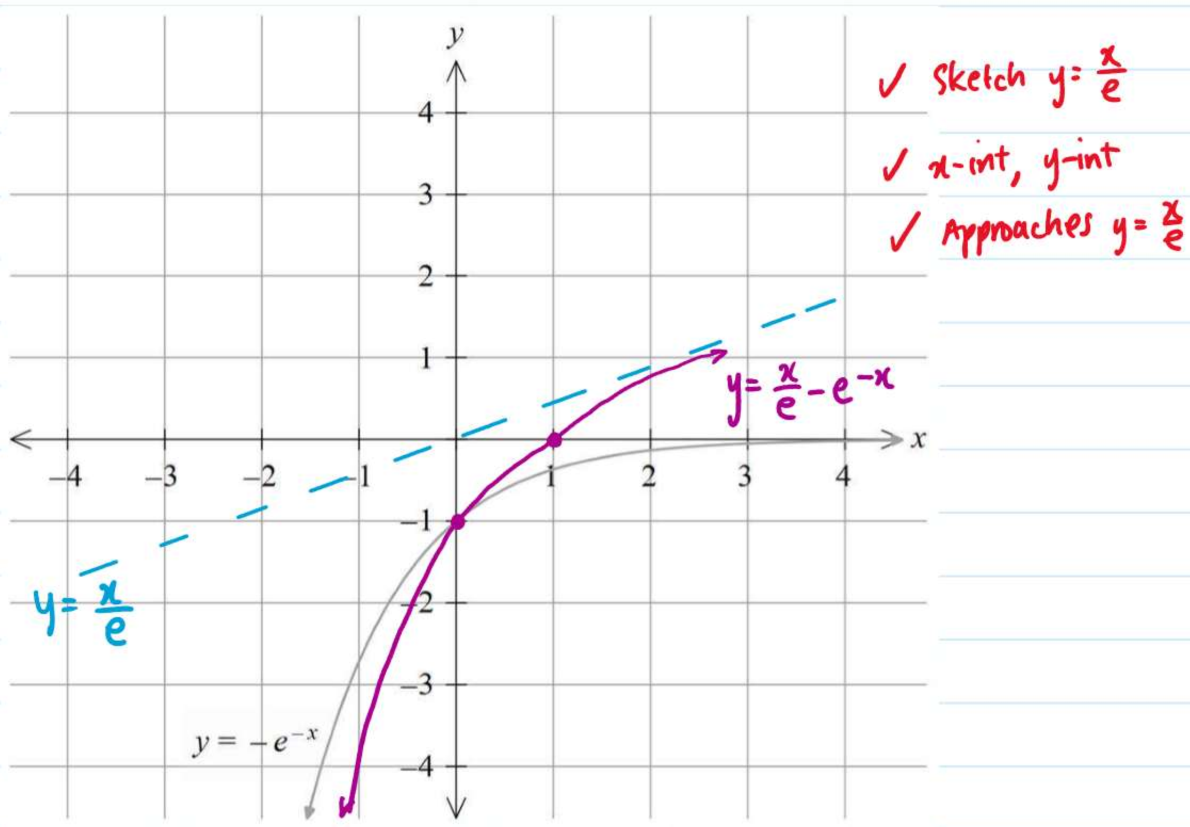
${}^8C_6 = 28$

NOT TOGETHER $= 56 + 56 + 28$

$= 140$ ✓

(d) $y = \frac{x}{e} - e^{-x}$

Consider $\begin{cases} y = x/e \\ y = -e^{-x} \end{cases}$



(c) There are six male and four female players in a mixed volleyball squad. Six players are on the court at any one time during play and others can be substituted when appropriate.

(i) In how many ways can the team on court be chosen if there are equal numbers of male and female players? **1**

(ii) In how many ways can the team on court be chosen such that a particular male and a particular female do not play on the court at the same time? **2**

(i) Mostly well done.

(ii) Poorly done. Students who considered cases were less likely to succeed. Some students attempted to use "Separate = Total – Together" but used 80 as the total, rather than $\binom{10}{6}$. Some students assumed that part ii) continued from part i), so they kept the restriction of 3 males and 3 females: $80 - \binom{5}{2} \times \binom{3}{2} = 50$ and they were awarded full marks for this as it was an understandable misunderstanding and did not simplify the question – students were able to demonstrate the skill being assessed.

(d) **On the Question 9 template** and using addition of ordinates, draw a neat sketch of the function below, labelling any intercepts with the coordinate axes. **3**

$$y = \frac{x}{e} - e^{-x}$$

Very poorly done. An alarmingly large percentage of students did NOT sketch the straight line $y = \frac{x}{e}$, which is concerning on multiple levels: The question asks students to use *addition of ordinates*, which cannot be done without both graphs being first plotted on the axes!!

It is not a hyperbola, students mistaking it for $y = \frac{e}{x}$ (and some sketching $y = \frac{e}{x}$ incorrectly anyway)

Students who did not sketch $y = \frac{x}{e}$ were only able to achieve at most 1 mark, since they would not have been able to show the asymptotic behaviour of $y = \frac{x}{e} - e^{-x}$ as $x \to \infty$ without the line there to approach.

Students who had the correct x- and y-intercepts but wrong 'shape' of the graph did not gain any marks.

Question 10

(a) $4\times L$, $9\times I$, $8\times S$, $14\times A$, $3\times V$

Cannot get $6\times L$ or $6\times V$, so 31 tiles left.

$\{I, S, A\} \Rightarrow 3$ pigeonholes

$\left\lceil \frac{n}{3} \right\rceil = 6$

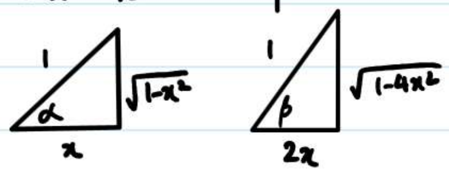
$n = 5\times 3 + 1 = 16$ ✓

BUT Lisa could pull all L and V tiles first

$\therefore 16 + 7 = \underline{23}$ ✓

(b) $\underbrace{\cos^{-1} x}_{\alpha} - \underbrace{\cos^{-1}(2x)}_{\beta} = \cos^{-1}(2x^2)$

$\cos\alpha = x$ $\cos\beta = 2x$



✓

$\alpha - \beta = \cos^{-1}(2x^2)$

$2x^2 = \cos(\alpha - \beta)$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \sqrt{1-x^2} \times \sqrt{1-4x^2} + x \times 2x$

$= \sqrt{(1-x^2)(1-4x^2)} + 2x^2$

$0 = \sqrt{(1-x^2)(1-4x^2)}$

$(1-x)(1+x)(1-2x)(1+2x) = 0$

$x = \pm 1, \pm\frac{1}{2}$ ✓

BUT $\cos^{-1}(2x)$ and $\cos^{-1}(2x^2)$ are undefined for $x = \pm 1$

$\therefore x = \pm\frac{1}{2}$ only ✓

Question 10 (11 marks) Use a SEPARATE writing booklet.

(a) Lisa has a bag of random letter tiles. The bag contains four L tiles, nine I tiles, eight S tiles, fourteen A tiles and three V tiles. **2**

What is the minimum number of tiles she can pull out of the bag to ensure she gets six of the same letter?

Mostly well done. Students should mention the Pigeonhole Principle

(b) Solve for x **3**

$$\cos^{-1} x - \cos^{-1}(2x) = \cos^{-1}(2x^2)$$

Done well by about half the cohort. A common misconception was thinking if $x = \cos^{-1}\alpha$ and $\alpha = \cos x$ then $2x^2 = \cos^{-1}(2x^2)$, which it does not, $2x^2 = 2(\cos^{-1} x)^2$. As a result, students co-incidentally answered $x = \frac{1}{2}$.

Full marks awarded only if students stated that $x = \pm 1$ is not defined in the equation. Some students did not include both $x = \pm\frac{1}{2}$.

(c) $f(x) = 6x^3 - 5x^2 - 7x + 4$

(i) $f(-1) = 6(-1)^3 - 5(-1)^2 - 7(-1) + 4$

$= 0$

$\therefore (x+1)$ is a factor ✓

$$\begin{array}{r} 6x^2 - 11x + 4 \\ x+1 \overline{) 6x^3 - 5x^2 - 7x + 4} \\ 6x^3 + 6x^2 \\ \hline -11x^2 - 7x \\ -11x^2 - 11x \\ \hline 4x + 4 \\ 4x + 4 \\ \hline 0 \end{array}$$

$f(x) = (x+1)(6x^2 - 11x + 4)$ ✓

$= (x+1)(3x-4)(2x-1)$ ✓

(ii) $5\cos 2\theta + 3 = 2(\sin\theta + 6\sin\theta\cos^2\theta)$ $\quad 0 \le x \le 2\pi$

$5(1 - 2\sin^2\theta) + 3 = 2(\sin\theta + 6\sin\theta(1 - \sin^2\theta))$

$5 - 10\sin^2\theta + 3 = 2(\sin\theta + 6\sin\theta - 6\sin^3\theta)$ ✓

$8 - 10\sin^2\theta = 14\sin\theta - 12\sin^3\theta$

$12\sin^3\theta - 10\sin^2\theta - 14\sin\theta + 8 = 0$

$6\sin^3\theta - 5\sin^2\theta - 7\sin\theta + 4 = 0$

$(\sin\theta + 1)(3\sin\theta - 4)(2\sin\theta - 1) = 0$ from (i) ✓

$\sin\theta = -1$ $\qquad$ $\sin\theta = 4/3$ $\qquad$ $\sin\theta = 1/2$

$\theta = \frac{3\pi}{2}$ $\qquad$ No soln $\qquad$ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ ✓

(c) A cubic polynomial is defined by $f(x) = 6x^3 - 5x^2 - 7x + 4$.

(i) Express $f(x)$ as a factor of three linear factors. **3**

(ii) Hence, solve for $0 \le \theta \le 2\pi$ **3**

$$5\cos 2\theta + 3 = 2(\sin\theta + 6\sin\theta\cos^2\theta)$$

Leave your answers in exact form.

(i) Done well by many whether done by inspection and division or sum and products of roots.

(ii) The word hence should have given students clues about the strategy for this question. Part (i) is a one variable question so altering part (ii) into one ratio (sin is clearly the easier choice) would mean a substitution of $x = \sin\theta$, that will make solving much easier. Students that got that far are picking up well the undefined solution $\sin\theta = \frac{4}{3}$. Some issues with expanding and simplifying, hence, means there is a connection so if your equation isn't looking like part (i) then you should check the working.