Name: ____________________

# CARINGBAH HIGH SCHOOL

**2020** | **Preliminary Task 3**

# Mathematics Extension 1

**General Instructions**

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using black pen
- NESA approved calculators and Mathomat may be used
- A reference sheet is provided
- In Questions 11–15, show relevant mathematical reasoning and/or calculations

**Total marks: 65**

**Section I – 10 marks** (pages 2–4)

- Attempt Questions 1–10
- Allow about 10 minutes for this section

**Section II – 55 marks** (pages 5–9)

- Attempt Questions 11–15
- Allow about 80 minutes for this section

Marker's Use Only

| Question | 1 – 10 | 11 | 12 | 13 | 14 | 15 | Total | |
|---|---|---|---|---|---|---|---|---|
| Mark | /10 | /11 | /11 | /11 | /11 | /11 | /65 | % |

**This paper must not be removed from the examination room**

## Section I

**10 marks**
**Attempt Questions 1–10**
**Allow about 10 minutes for this section**

Use the multiple-choice answer sheet for Questions 1–10.

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**1** Which of the following is a factor of $P(x) = x^3 + x^2 + x + 1$?

(A) $(x - 1)$

(B) $(x + 1)$

(C) $\left(x - \frac{1}{2}\right)$

(D) $\left(x + \frac{1}{2}\right)$

**2** What are the integers $x$ and $y$ such that $\frac{\sqrt{3}}{2\sqrt{3}+3} = x + y\sqrt{3}$ ?

(A) $x = -1$ and $y = 2$

(B) $x = 1$ and $y = -2$

(C) $x = 2$ and $y = -1$

(D) $x = -2$ and $y = 1$

**3** A six-sided die has one side with a value of 1, two sides each with a value of 2 and three sides each with a value of 3. On rolling this die what would be the expected value?

(A) $2$

(B) $2\frac{1}{3}$

(C) $2\frac{1}{2}$

(D) $2\frac{2}{3}$

**4** If $f(x) = x^2$ and $g(x) = \sqrt{x}$. Find $g\big(f(2)\big)$.

(A) $x$

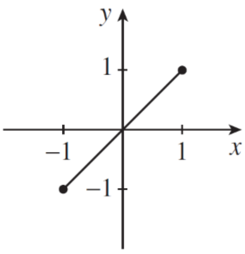
(B) 0

(C) 1

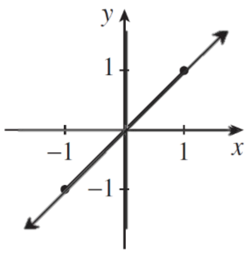
(D) 2

**5** Which of the following is the graph of $y = \sin^{-1}(\sin x)$ ?

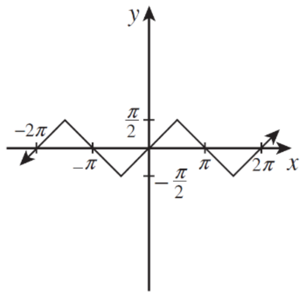
(A)



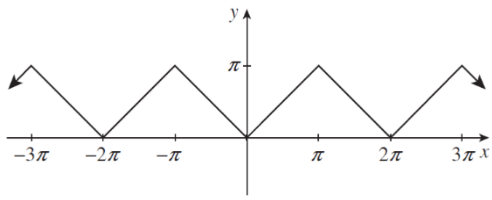

(B)




(C)




(D)




**6** Which expression is equivalent to $\frac{2\cos^2\theta}{1-\sin\theta}$ ?

(A) $2 - 2\sin\theta$

(B) $2 + 2\sin\theta$

(C) $2 - 2\cos\theta$

(D) $2 - 2\cos\theta$

**7** For which value of $a$ is this function continuous for all values of $x$?

$$f(x) = \begin{cases} a - x^2\ , & x < 1 \\ (x-a)^2, & x \geq 1 \end{cases}$$

(A) $a = 5$

(B) $a = 4$

(C) $a = 3$

(D) $a = 2$

**8** What is the domain of the function $f(x) = \ln(3x - x^2)$?

(A) $0 < x < 3$

(B) $0 \leq x < 3$

(C) $0 < x \leq 3$

(D) $0 \leq x \leq 3$

**9** Which parametric equations represent a circle?

(A) $x = 2t$ and $y = t^2$

(B) $x = 2t$ and $y = t + 2$

(C) $x = 2\cos(t)$ and $y = \sin(t)$

(D) $x = 2\cos(t) + 2\sin(t)$ and $y = 2\cos(t) - 2\sin(t)$

**10** Three girls and three boys sit around a round table.

What is the probability that they sit boy-girl-boy-girl-boy-girl?

(A) $\frac{1}{10}$

(B) $\frac{3}{20}$

(C) $\frac{1}{120}$

(D) $\frac{1}{20}$

## Section II

**55 marks**
**Attempt Questions 11–15**
**Allow about 80 minutes for this section**

Answer each question on SEPARATE writing paper. Extra writing booklets are available.

In Questions 11 – 15, your responses should include relevant mathematical reasoning and/or calculations.

**Marks**

**Question 11** (11 marks) Use SEPARATE writing paper.

(a) Solve: **2**

$$|2x - 1| \geq 3$$

(b) A bag contains three red, four yellow and five blue balls. If three balls are drawn from the bag simultaneously, find the probability that:

(i) all three balls are red. **1**

(ii) At least one of the balls are yellow? **2**

(c) If $\sin x = \frac{2}{3}$, where $90° \leq x \leq 180°$ and $\cos y = \frac{3}{4}$, where $0° \leq y \leq 90°$, what is the exact value of $\sin(x + y)$? **3**

(d) If $P(x) = x(x^2 - 1)^2$,

(i) Find all the roots of $P(x)$. **1**

(ii) Show that $P(1) = P'(1) = 0$. **2**

**End of Question 11**

**Marks**

**Question 12** (11 marks) Use SEPARATE writing paper.

(a) Sketch the graph of $y = (1-x)^3(2+x)^2(3-x)$ showing all the intercepts. **3**

(b) Let $f(x) = (x-1)^2$ , for the domain $x > 1$.

(i) Classify $f(x)$ as one-to-one or many-to-one. **1**

(ii) Find any values of $x$ for which $f(x) = f^{-1}(x)$ . **2**

(c) Find the multiplicity of the root $x = -1$ of the equation $P(x) = 0$ where $P(x) = x^4 + 4x^3 + 7x^2 + 6x + 2$. **2**

(d) A certain chemical treatment of blue-green algae in a river causes it to decrease based on the equation $A = A_0 \times e^{kt}$, where $A$ is the kilograms of algae and $t$ is time in months. **3**

If 250 kg of blue-green algae reduces to 150 kg after 3 months, find how long it will take, to the nearest month, to reduce the bluegreen algae to 10 kg.

**End of Question 12**

**Marks**

**Question 13** (11 marks) Use SEPARATE writing paper.

(a) Sketch the graph of the function $f(x) = -2\tan^{-1}x$ showing the intercepts on the axes and the equations of the asymptotes. **2**

(b) A polynomial of degree 4 has a double zero at 2 with the other two zeros being $k$ and $-k$. When $x = 1$ it takes the value 8 and when $x = 3$ it takes the value 80. **3**
Find the possible values of $k$.

(c) If $f(x) = x^2 - 1$ sketch the graph of $y = \dfrac{1}{f(x)}$ indicating all intercepts and asymptotes. **3**

(d) A Maths test is to consist of ten questions.

(i) In how many ways can it be arranged so that the shortest question is first and the longest question is last? **1**

(ii) What is the probability that the shortest and longest questions are next to one another? **2**

**End of Question 13**

**Marks**

**Question 14** (11 marks) Use SEPARATE writing paper.

(a) The numbers 1 to 20 are either odd or even as wells as being prime, composite or neither. **2**
If an odd number is chosen at random what is the probability that it is also prime?

(b) i) Find a simplified expression for $\operatorname{cosec}\theta\,(\cos\theta - 1)$ in terms of $t$ where $t = \tan\frac{\theta}{2}$. **2**

ii) Hence solve $\operatorname{cosec}\theta\,(\cos\theta - 1) = -1$ for $0° \leq \theta \leq 180°$. **1**

(c) If $\alpha, \beta$ and $\gamma$ are the roots of the cubic equation $2x^3 + 4x - 3 = 0$. **3**

Find the value of $\alpha^3 + \beta^3 + \gamma^3$.

(d) Solve the inequation: **3**

$$\frac{x}{x-1} \geq 2$$

**End of Question 14**

**Marks**

**Question 15** (11 marks) Use SEPARATE writing paper.

(a) Solve: **3**

$$2 \times 4^x - 3 \times 2^x + 1 = 0.$$

(b) Find the number of ways in which the letters of the word TRIANGLE can be arranged in a line:

i) so that all the vowels are next to each other? **1**

ii) so that consonants are in the two end positions? **1**

(c) Consider the function $f(x) = (x-2)^2,\ \ 0 \le x \le 2$.

i) Find the inverse function $f^{-1}(x)$ and state its domain. **2**

ii) Sketch the graph of the inverse function $f^{-1}(x)$ showing the endpoints. **2**

(d) If $\sec x + \tan x = k$ , find an expression for $\sec x$ in terms of $k$. **2**

**End of Exam**

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**Name:** ____________________

# Mathematics Extension 1

## Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample** **2 + 4 = ?** **(A) 2 (B) 6 (C) 8 (D) 9**

**A ○ B ● C ○ D ○**

**If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.**

**A ● B ⊗ C ○ D ○**

**If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:**

***correct***

**A ⊗ B ⊗ C ○ D ○**

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Start Here ➔

| | | | | |
|---|---|---|---|---|
| **1.** | A ○ | B ○ | C ○ | D ○ |
| **2.** | A ○ | B ○ | C ○ | D ○ |
| **3.** | A ○ | B ○ | C ○ | D ○ |
| **4.** | A ○ | B ○ | C ○ | D ○ |
| **5.** | A ○ | B ○ | C ○ | D ○ |
| **6.** | A ○ | B ○ | C ○ | D ○ |
| **7.** | A ○ | B ○ | C ○ | D ○ |
| **8.** | A ○ | B ○ | C ○ | D ○ |
| **9.** | A ○ | B ○ | C ○ | D ○ |
| **10.** | A ○ | B ○ | C ○ | D ○ |

# Y11 Extension 1 2020 AT3 Exam Answers

## Section I

**Q1)** $P(-1) = -1^3 + -1^2 - 1 + 1 = 0$

$\therefore (x+1)$ is a factor. **(B)**

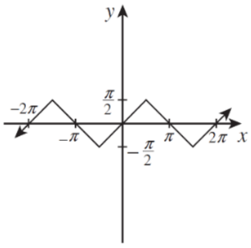
**Q2)** $\frac{\sqrt{3}}{2\sqrt{3}+3} \times \frac{2\sqrt{3}-3}{2\sqrt{3}-3} = \frac{6-3\sqrt{3}}{12-9} = \frac{3(2-\sqrt{3})}{3} = 2 - \sqrt{3}$

$\therefore x = 2\ and\ y = -1$ **(C)**

**Q3)** $E(X) = \left(1 \times \frac{1}{6}\right) + \left(2 \times \frac{2}{6}\right) + \left(3 \times \frac{3}{6}\right) = \frac{14}{6} = 2\frac{1}{3}$ **(B)**

**Q4)** $g(f(x)) = \sqrt{(x^2)} = x \ \therefore g(f(2)) = 2$ **(D)**

**Q5)** $y = \sin^{-1}(\sin x)$ **(C)**




**Q6)** $\frac{2\cos^2\theta}{1-\sin\theta} = \frac{2\,(1-\sin^2\theta)}{1-\sin\theta} = \frac{2\,(1-\sin\theta)(1+\sin\theta)}{1+\sin\theta}$

$= 2 + 2\sin\theta$ **(B)**

**Q7)** When $a = 2$ , $f(x) = 2 - x^2$ for $x < 1$ and

$(x-2)$^2 for $x \geq 1$

which is "smooth "& continuous **(D)**

**Q8)** Domain of $f(x) = \ln(3x - x^2)$ is $(0,3)$ **(A)**

**Q9)** Parametric equations of a circle are

$x = 2\cos(t) + 2\sin(t)\ \ \&\ y = 2\cos(t) - 2\sin(t)$ **(D)**

**Q10)** $P(\text{Boy} - \text{girl} - \text{boy} - \text{girl}) = \frac{3!\,2!}{5!} = \frac{1}{10}$ **(A)**

## Section II

**Q11)**

**a)** $|2x - 1| \geq 3 \ \therefore 2x - 1 \geq 3$ or $-2x + 1 \leq 3$

$\therefore x \geq 2\ or\ x \leq -1\ \ or\ (-\infty, -1] \cup [2, \infty)$

**b)**

i) $P(\text{all red}) = \frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} = \frac{1}{220}$

ii) $P(\text{at least 1 yellow}) = 1 - (\frac{8}{12} \times \frac{7}{11} \times \frac{6}{10}) = \frac{41}{55}$

**c)** $\sin x = \frac{2}{3}$, where $90° \leq x \leq 180°\ \therefore \cos x = \frac{-\sqrt{5}}{3}$

$\cos y = \frac{3}{4}$, where $0° \leq x \leq 90°\ \therefore \sin y = \frac{\sqrt{7}}{4}$

$\therefore \sin(x+y) = \frac{2}{3} \times \frac{3}{4} - \frac{\sqrt{5}}{3} \times \frac{\sqrt{7}}{4} = \frac{6-\sqrt{35}}{12} = \frac{1}{2} - \frac{\sqrt{35}}{12}$

**d)**

i) $P(x) = x(x^2 - 1)^2 \ \therefore$ Roots $x = -1, 0, 1$

ii) $P(1) = 1(1^2 - 1)^2 = 0$

$P'(x) = (x^2-1)^2 \times 1 + x \times 2(x^2-1)^1 \times 2x$

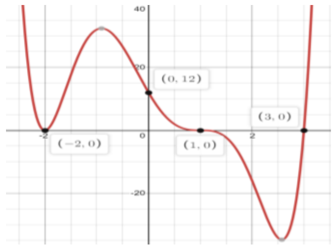
$= (x^2 - 1)[(x^2 - 1) + 4x^2]$

$\therefore P'(1) = (1^2 - 1)[(1 - 1) + 4 \times 1^2\ ] = 0$

$\therefore P(1) = P'(1) = 0$

**Q12)**

**a)** $y = (1-x)^3(2+x)^2(3-x)$




**b)** $f(x)\ = 2(x-1)^2$ for the domain $x > 1$

i) One-to-One

ii) $f(x) = f^{-1}(x)$ when $f(x) = x$

$\therefore 2(x-1)^2 = x$

$\therefore 2x^2 - 4x + 2 = x$

$\therefore 2x^2 - 5x + 2 = 0$

$\therefore (2x - 1)(x - 2) = 0$

$\therefore x = 2$ only for the domain $x > 1$

**c)** $P(x) = x^4 + 4x^3 + 7x^2 + 6x + 2$

$\therefore P(-1) = -1^4 + 4.-1^3 + 7.-1^2 + 6.-1 + 2 = 0$

therefore $(x+1)$ is a factor

$\therefore P'(x) = 4x^3 + 12x^2 + 14x + 6$

$\therefore P'(-1) = 4.-1^3 + 12.-1^2 + 14.-1 + 6 = 0$

therefore $(x+1)^2$ is a factor

$\therefore P''(x) = 12x^2 + 24x + 14$

$\therefore P''(-1) = 12.-1^2 + 24.-1 + 14 \neq 0$

therefore multiplicity is 2

**d)** $A = 250 \times e^{kt}$

when $t = 3, A = 150 = 250 \times e^{k\times 3}$

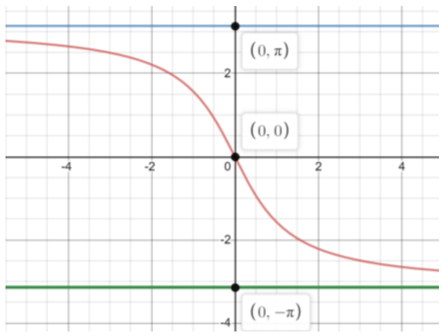
therefore $k = \ln(\frac{150}{250}) \div 3 \approx -0.17$

when $A = 10 = 250 \times e^{-0.17t}$ ,

$t = \ln(\frac{10}{250}) \div -0.17 \approx 19$ months

**Q13)**

**a)** $f(x) = -2\tan^{-1} x$



# Y11 Extension 1 2020 AT3 Exam Answers

**Q13) (Continued)**

**b)** $P(x) = a(x-2)^2(x-k)(x+k)$

$P(1) = a(1-2)^2(1-k)(1+k) = 8$

$\therefore a(1-k^2) = 8 \;\; - ①$

$P(3) = a(3-2)^2(3-k)(3+k) = 80$

$\therefore a(9-k^2) = 80 \;\; - ②$

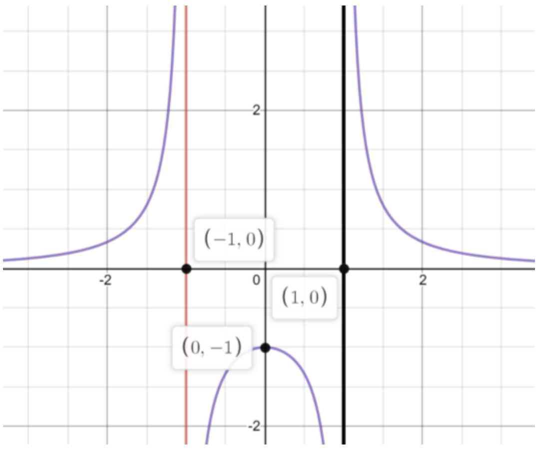
$\therefore ② \div ① \;\; \frac{a(9-k^2)}{a(1-k^2)} = 10$

$\therefore (9-k^2) = 10\,(1-k^2)$

$\therefore 9 - k^2 = 10 - 10k^2$

$\therefore 9k^2 = 1\,,\;\; k = \pm\frac{1}{3}$

**c)** $f(x) = x^2 - 1 \;\; \therefore y = \frac{1}{f(x)}$




**d)**

i) $1 \times 1 \times 8! = 40{,}320$

ii) $P = \frac{9!\,2!}{10!} = \frac{1}{5}$

**Q14)**

**a)** odds $= 10$ primes which are odd $= 7$

$\therefore P = \frac{7}{10}$

**b)**

i) $\operatorname{cosec}\theta\,(\cos\theta - 1) = \frac{1+t^2}{2t} \times \left(\frac{1-t^2}{1+t^2} - 1\right)$

$= \frac{1+t^2}{2t} \times \left(\frac{1-t^2}{1+t^2} - \frac{1+t^2}{1+t^2}\right) = \frac{1+t^2}{2t} \times \left(\frac{1-t^2-1-t^2}{1+t^2}\right)$

$= \frac{1+t^2}{2t} \times \left(\frac{-2t^2}{1+t^2}\right) = -t$

ii) $\operatorname{cosec}\theta\,(\cos\theta - 1) = -1 \;\; \therefore -t = -1$

$\therefore t = \tan\frac{\theta}{2} = 1 \;\; \therefore \; \frac{\theta}{2} = 45° \;\; \therefore \; \theta = 90°$

**c)** $2x^3 + 4x - 3 = 0 \;\; \therefore 2\alpha^3 + 4\alpha - 3 = 0$

$\therefore 2\alpha^3 = -4\alpha + 3$ and similar for $\beta^3$ and $\gamma^3$

$\therefore \alpha^3 + \beta^3 + \gamma^3 = (-4\alpha + 3 - 4\beta + 3 - 4\gamma + 3)/2$

$= (-4(\alpha+\beta+\gamma) + 9)/2 = (-4(0)+9)/2 = 9/2$

**Q14) (Continued)**

**d)** $\frac{x}{x-1} \geq 2, \therefore 2(x-1) \leq x, \therefore x - 2 \leq 0,$

$\therefore x \leq 2 \;\&\; x \neq 1 \;\; \therefore 1 < x \leq 2$

**Q15)**

**a)** $2 \times 4^x - 3 \times 2^x + 1 = 0$

$\therefore 2 \times (2^2)^x - 3 \times 2^x + 1 = 0$

$\therefore 2 \times (2^x)^2 - 3 \times 2^x + 1 = 0$ , Let $u = 2^x$

$\therefore 2u^2 - 2u + 1 = 0, \therefore (2u-1)(u-1) = 0$

$\therefore u = 2^x = \frac{1}{2}$ or $1, \therefore x = -1$ or $0$

**b)** TRIANGLE

i) all the vowels next to each other:

$2! \times 3! \times 5! = 1440$

ii) consonants are in the two end positions:

Choose the first and last letters from the 5 consonants, then arrange the remaining 6 letters between them in $(5 \times 4) \times 6! = 14400$

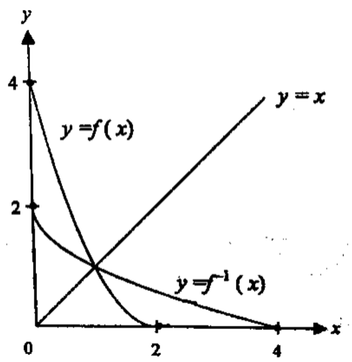
**c)** $f(x) = (x-2)^2, \;\; 0 \leq x \leq 2$

**i)** $y = (x-2)^2, \;\; 0 \leq x \leq 2$ and $0 \leq y \leq 4$

$\therefore \sqrt{y} = |x-2| \;\; \therefore \sqrt{y} = 2 - x \;\; \therefore x = 2 - \sqrt{y}$

$\therefore f^{-1}(x) = 2 - \sqrt{x}$ , $\therefore$ Domain $0 \leq x \leq 4$ $[0,4]$

ii)




**d)** $\sec x + \tan x = k \;\;\; - ①$

$1 + \tan^2 x = \sec^2 x$

$\therefore \sec^2 x - \tan^2 x = 1$

$\therefore (\sec x + \tan x)\,(\sec x - \tan x) = 1$

$\therefore k \times (\sec x - \tan x) = 1$

$\therefore \sec x - \tan x = \frac{1}{k} \;\;\; - ②$

$\therefore \; ① + ② \; = \; 2\sec x = k + \frac{1}{k}$

$\therefore \; \sec x = \frac{1}{2}\left(k + \frac{1}{k}\right)$