BAULKHAM HILLS HS PERSEVERE

2025 **PRELIMINARY YEARLY EXAMINATION**

Mathematics Extension

General Instructions	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • In Questions 11 – 14, show relevant mathematical reasoning and/or calculations
Total marks: 70	**Section I – 10 marks** (pages 2 – 5) • Attempt Questions 1 – 10 • Allow about 15 minutes for this section **Section II – 60 marks** (pages 6 – 10) • Attempt Questions 11 – 14 • Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

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1 The polynomial $2x^3+6x^2-7x-10$ has zeros α, β and γ. What is the value of $\alpha\beta\gamma(\alpha+\beta+\gamma)$?

A. -60

B. -15

C. 15

D. 60

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2 Which of the following best describe the solution to the given equation $|x-5|=2x$?

A. 5

B. $\frac{5}{3}$

C. -5 and $\frac{5}{3}$

D. 5 and $\frac{5}{3}$

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3 In how many ways can 7 people be chosen from a group of 16 people and then arranged in a circle?

A. $\frac{15!}{9!}$

B. $\frac{15!}{9!7!}$

C. $\frac{16!}{9!}$

D. $\frac{16!}{9!7}$

4 Which of the following is the coefficient of the fourth term in the expansion of $(3x-4)^6$?

A. -34560

B. 34560

C. 25920

D. -25920

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5 Which of the following values of x could satisfy $2\sin^{-1}x+\cos^{-1}x=\dfrac{\pi}{3}$?

A. $-\dfrac{1}{2}$

B. $\dfrac{1}{2}$

C. $\dfrac{\sqrt{3}}{2}$

D. $-\dfrac{\sqrt{3}}{2}$

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6 What is the range of the function $y=\tan^{-1}(\sin x)$?

A. $y\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$

B. $y\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$

C. $y\in\left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right]$

D. $y\in\left(-\dfrac{\pi}{4},\dfrac{\pi}{4}\right)$

7 Which of the following is an expression for $\frac{dy}{dx}$ if $x = 2\left(t + \frac{1}{t}\right)$ and $y = 2\left(t - \frac{1}{t}\right)$?

A. $\frac{t^2+1}{t^2-1}$

B. $\frac{t+1}{t-1}$

C. $\frac{t^2-1}{t^2+1}$

D. $\frac{t-1}{t+1}$

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8 In triangle ABC, if $\cos A = \frac{1}{3}$, then which of the following equals to $\cot\left(\frac{A}{2}\right)$?

A. $\sqrt{2}$

B. $-\sqrt{2}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{2}}$

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9 The polynomial $P(x) = x^3 + 5x^2 + x - 2$ has three real zeros. If $P(x)$ is translated to the right by 1 unit, what is the product of the zeros of the transformed polynomial?

A. -5

B. -1

C. 1

D. 5

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10 The sum of the solutions of $\sin(2x) = \frac{\sqrt{3}}{2}$ over the interval $[-\pi, d]$ is $-\pi$. What could be the value of *d*?

A. 0

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{7\pi}{6}$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate section of the writing booklet. Extra writing paper is available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (16 marks)

a) A scholarship committee needs to assign one of 4 ranks (Gold, Silver, Bronze, Participation) to each student. What is the minimum number of students required to ensure that at least one rank is given to 6 students? **1**

b) Show that $\dfrac{\cos 23^o - \sqrt{3}\sin 23^o}{2} = \cos 83^o$. **2**

c) The polynomial $P(x) = x^3 - 4x^2 + kx + 10$ has two zeros that are the same in magnitude but opposite in sign. Find the value of k and all zeros of the polynomial. **3**

d) A curve is defined by the following parametric equations: **3**

$$x = -3 + 2\cos^2\theta \text{ and } y = -5 + \cos 2\theta$$

Find the Cartesian equation of the curve, noting any restrictions.

e) If α, β and γ are roots of $2x^3 - 4x^2 + 1 = 0$, find the value of:

i) $\alpha^2 + \beta^2 + \gamma^2$ **2**

ii) $\dfrac{\alpha}{\beta\gamma} + \dfrac{\beta}{\alpha\gamma} + \dfrac{\gamma}{\alpha\beta}$ **1**

Question 11 continues next page

f) Consider the function $f(x) = 2\cos^{-1} x$

i) State the domain and range of $y = f(x)$. **2**

ii) Sketch the graph $y = \left|2\cos^{-1} x - \pi\right|$ **2**

End of Question 11

Question 12 (15 marks)

a) The polynomial $P(x)$ is given by $P(x) = x^3 - (k-1)x^2 + (1-k)x + 1$ for some real number k.

i) Show that $x = -1$ is a root of the equation $P(x) = 0$. **1**

ii) Given that $P(x) = (x+1)(x^2 - kx + 1)$, find the set of values of k such that the equation $P(x) = 0$ has three distinct real roots. **3**

b) A bowl of hot soup at temperature $T^o C$, is placed in a cooler environment. It loses heat according to the law $\frac{dT}{dt} = k(T - T_0)$, where k us constant of proportionality, t is the time elapsed in minutes, and T_0 is the temperature of the environment in degrees Celsius.

i) Show that $T = T_0 + Ae^{kt}$ satisfies the above equation, where A is constant. **1**

ii) A bowl of soup at 96°C is left to stand in a room at a temperature of 18°C. After 3 minutes the soup cools down to 75°C. Calculate the value of k correct to 4 decimal places. **2**

iii) Susan wishes to enjoy her soup at a temperature of 60°C. How long should she wait? Answer to the nearest minutes. **2**

Question 12 continues next page

c) A total of five players is selected at random from four sporting teams. Each of the teams consists of ten players numbered from 1 to 10.

i) What is the probability that of the five selected players, three are numbered 4 and two are numbered 6? **2**

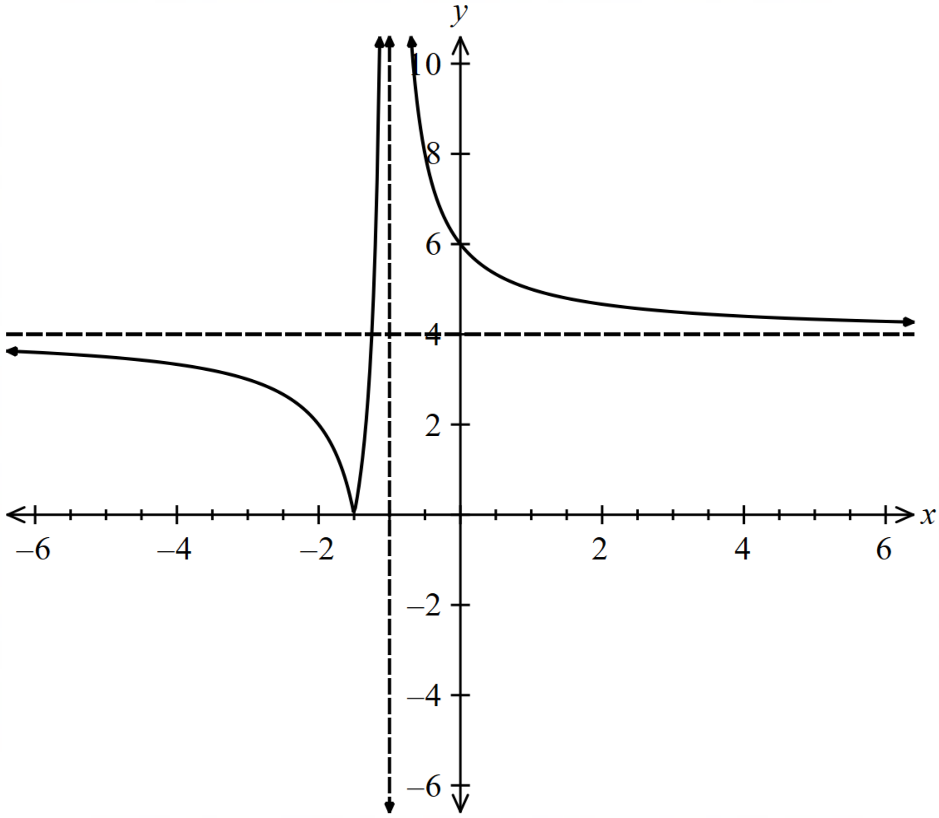
ii) What is the probability that the five selected players contain at least four players from the same team? **2**

d) Prove that $\tan^{-1}\frac{2}{3}+\cos^{-1}\frac{2}{\sqrt{5}}=\tan^{-1}\frac{7}{4}$. **2**

End of Question 12

Question 13 (15 marks)

a) The diagram below is a sketch of the graph of the function $y = |f(x)|$, where $f(x) = \frac{2}{x+1} + 4$ and horizontal asymptote at $y = 4$.



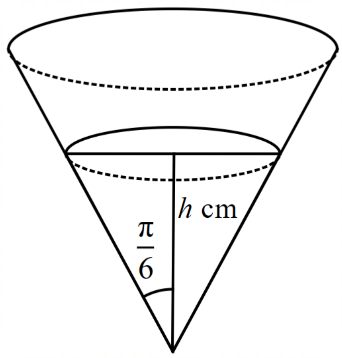
On separate diagrams, sketch the following graphs, in each case showing the intercepts on the axes and any asymptotes.

i) $|y| = f(|x|)$ **2**

ii) $y = \sqrt{f(x)}$ **2**

Question 13 continues next page

b) An egg timer in the shape of an inverted right circular cone with semi-vertical angle $\frac{\pi}{6}$ radians. It contains sand to a depth of h cm as shown in the diagram below, which flows out of the apex of the cone at a constant rate of 0.5 cm^3/s.



i) Show that the volume V cm^3 of sand in the cone is given by $V = \frac{1}{9}\pi h^3$ **1**

ii) Find the value of h when the depth of sand in the egg timer is decreasing at a rate of 0.05 cm/s, giving your answer correct to 2 decimal places. **3**

c) Given that $\sin x + \cos x = k$, find the value of $\sin(2x)$ in terms of k. **2**

d) Consider the function $f(x) = ax^3 + bx^2 + cx + d$, where a, b, c and d are real numbers.

i) Write down the expression for $f'(x)$. **1**

ii) Given that the inverse of $f(x)$ is not a function, use part i) to show that $b^2 - 3ac > 0$. **2**

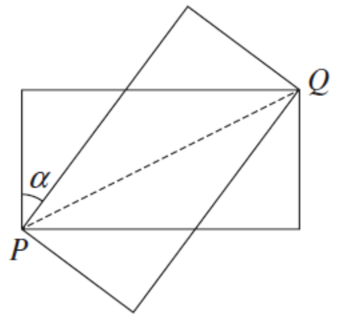
iii) Consider $g(x) = \frac{1}{2}x^3 - 3x^2 + 6x - 8$, use the result in part (ii) to show that the inverse of $g(x)$ is a function. **2**

End of Question 13

Question 14 (14 marks)

a) Solve for x: $|x-1| > 2\sqrt{x(1-x)}$ **3**

b) Two identical rectangles share a common diagonal PQ as shown in the following diagram. For each rectangle, the ratio of the longer side to the shorter side is x : 1. The angle at P between the sides of two rectangles is α. If $\alpha = \dfrac{\pi}{3}$ radians, then show that $x = \sqrt{3} + 2$. **3**



c) i) Show that $x^n(1+x)^n\left(1+\dfrac{1}{x}\right)^n = (1+x)^{2n}$ **2**

ii) Hence show that $1 + \left({}^nC_1\right)^2 + \left({}^nC_2\right)^2 + \ldots + \left({}^nC_n\right)^2 = {}^{2n}C_n$ **3**

d) Given that $\dfrac{\sin\beta}{\sin(2\alpha+\beta)} = \dfrac{m}{n}$, prove that $\dfrac{n-m}{n+m}\tan(\alpha+\beta) = \tan\alpha$. **3**

End of Exam

Preliminary Yearly Task - 2025, Mathematics Extension-1

Marking Guidelines

Section - I

Multiple Choice (1 - 10)

1) B
2) B
3) D
4) A
5) A
6) A
7) A
8) C
9) B
10) C

Sample Solutions:

1) $2x^3 + 6x^2 - 7x - 10$ has zeros α, β and γ.

$\therefore \alpha + \beta + \gamma = -\frac{6}{2} = -3$ & $\alpha\beta\gamma = \frac{-(-10)}{2} = 5$

$\therefore \alpha\beta\gamma(\alpha + \beta + \gamma) = 5(-3)$

$= -15$ $\therefore$ B

2) $|x - 5| = 2x$; Condition is $x > 0$

$x - 5 = \pm 2x$

$\therefore x \mp 2x = 5$

$-x = 5$ or $3x = 5$

$x = -5$ or $x = \frac{5}{3}$

Since $x > 0$, $\therefore x = \frac{5}{3}$ is only solution. $\therefore$ B

3) ${}^{16}C_7$ then arrange 7 people in circle

$\therefore {}^{16}C_7 \; 6!$

$\frac{16!}{7!\,9!} \times 6! = \frac{16!}{9! \times 7}$ $\therefore$ (D)

4) $(3x-4)^6$

General term: $T_{r+1} = {}^nC_r \, x^{n-r} y^r$ for the expansion of $(x+y)^n$

For fourth term sub $r=3$

$\therefore T_4 = {}^6C_3 (3x)^3 (-4)^3$ $\therefore$ (A)

Coefficient of 4th term : ${}^6C_3 (3)^3 (-4)^3 = -34560$.

5) $2\sin^{-1}x + \cos^{-1}x = \frac{\pi}{3}$

$\sin^{-1}x + \frac{\pi}{2} = \frac{\pi}{3}$ $\left(\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$

$\sin^{-1}x = \frac{\pi}{3} - \frac{\pi}{2}$

$= -\frac{\pi}{6}$

$x = \sin\left(-\frac{\pi}{6}\right)$

$x = -\sin\frac{\pi}{6} = -\frac{1}{2}$ $\therefore$ (A)

6) $\cos A = \frac{1}{3}$ Let $t = \tan\frac{A}{2}$

$$\frac{1-t^2}{1+t^2} = \frac{1}{3}$$

$$3 - 3t^2 = 1 + t^2$$

$$4t^2 = 2$$

$$t^2 = \frac{2}{4}$$

$$t = \frac{1}{\sqrt{2}} \quad (\because t > 0)$$

$$\therefore \tan\frac{A}{2} = \frac{1}{\sqrt{2}}$$

$$\cot\frac{A}{2} = \sqrt{2}$$

$\therefore$ A

7) $x = 2\left(t + \frac{1}{t}\right)$, $y = 2\left(t - \frac{1}{t}\right)$

$$\frac{dx}{dt} = 2\left(1 - \frac{1}{t^2}\right) \qquad \frac{dy}{dt} = 2\left(1 + \frac{1}{t^2}\right)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$= \frac{2\left(1+\frac{1}{t^2}\right)}{2\left(1-\frac{1}{t^2}\right)}$$

$$= \frac{2(t^2+1)/t^2}{2(t^2-1)/t^2}$$

$$= \frac{t^2+1}{t^2-1}$$

$\therefore$ A

8) $y = \tan^{-1}(\sin x)$

Since $-1 \leq \sin x \leq 1$

$\therefore \tan^{-1}(-1) \leq \tan^{-1}(\sin x) \leq \tan 1$

$-\frac{\pi}{4} \leq \tan^{-1}(\sin x) \leq \frac{\pi}{4}$ $\therefore$ C

9) $P(x) = x^3 + 5x^2 + x - 2$

$x \rightarrow x-1$ as $P(x)$ translated 1 unit to the right.

$\therefore$ Translated Polynomial is

$(x-1)^3 + 5(x-1)^2 + (x-1) - 2$

$x^3 - 3x^2 + 3x - 1 + 5(x^2 - 2x + 1) + (x-1) - 2$

$x^3 - 3x^2 \ldots \underbrace{-1 + 5(1) - 1 - 2}_{\text{constant term.}}$

$\therefore$ Product of the zeros of translated poly. is

$-\dfrac{\text{Constant term}}{\text{Coeff. of } x^3}$

$-\frac{1}{1} = -1$ $\therefore$ B

10) $\sin 2x = \frac{\sqrt{3}}{2}$; $-\pi \le x \le \alpha$ for $-2\pi \le 2x \le 2\alpha$

$2x = -\pi - \frac{\pi}{3}, -2\pi + \frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, 2\pi + \frac{\pi}{3}, \ldots$

$2x = -\frac{4\pi}{3}, -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \ldots$

$x = -\frac{2\pi}{3}, -\frac{5\pi}{6}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \ldots$

Option A : Not possible as for $\alpha = 0$

Sum of roots $= -\frac{2\pi}{3} + -\frac{5\pi}{6} = -\frac{3\pi}{2} \ne -\pi$

Option B : Not possible as for $\alpha = \frac{\pi}{6}$

Sum of roots $= -\frac{2\pi}{3} - \frac{5\pi}{6} + \frac{\pi}{6} = -\frac{4\pi}{3} \ne -\pi$

Option C : for $\alpha = \frac{\pi}{3}$

Sum of roots $= -\frac{2\pi}{3} - \frac{5\pi}{6} + \frac{\pi}{6} + \frac{\pi}{3} = -\pi$

$\therefore$ (C)

END OF SECTION - 1

11 a) There are 4 ranks (Gold, Silver, Bronze, participation)

At least one rank is given to 6 students means

$4 \times 5 + 1 = 21$ 1 Correct solution

$\therefore$ Minimum number of students will be 21.

b) Show that $\dfrac{\cos 23^\circ - \sqrt{3}\sin 23^\circ}{2} = \cos 83^\circ$.

R.H.S $= \cos 83^\circ$ 2 Correct proof.

$= \cos(60^\circ + 23^\circ)$ 1 Correctly Identify

$= \cos 60^\circ \cos 23^\circ - \sin 60^\circ \sin 23^\circ$ $\cos 83^\circ = \cos(60 + 23^\circ)$

$= \frac{1}{2}\cos 23^\circ - \frac{\sqrt{3}}{2}\sin 23^\circ$ and apply $\cos(A+B)$

$= \dfrac{\cos 23^\circ - \sqrt{3}\sin 23^\circ}{2}$

$=$ L.H.S

c) $P(x) = x^3 - 4x^2 + kx + 10$.

Roots are: $\alpha, -\alpha, \beta$

$\sum \alpha = \alpha + (-\alpha) + \beta = 4 \Rightarrow \beta = 4$. —— (1)

$\sum \alpha\beta = \alpha(-\alpha) - \alpha\beta + \alpha\beta = k$

$-\alpha^2 = k$ —— (2)

$\prod \alpha = \alpha(-\alpha)\beta = -10$

$-\alpha^2\beta = -10$

$4k = -10$ (from (1) & (2))

$k = -5/2$

$\therefore \alpha^2 = \frac{5}{2}$

$\alpha = \pm\sqrt{\frac{5}{2}}$

$\therefore$ Roots are $\sqrt{\frac{5}{2}}, -\sqrt{\frac{5}{2}}, 4$

and $k = -\frac{5}{2}$

3 Correct solution

2 Find correct value of k and one of the root.

1 Correctly apply sum or product of the roots.

11d) $x = -3 + 2\cos^2\theta$ & $y = -5 + \cos 2\theta$

$2\cos^2\theta = x + 3$ $\quad\quad$ $\cos 2\theta = y + 5$

Since $\cos 2\theta = 2\cos^2\theta - 1$

2 Correct solution

$\therefore y + 5 = x + 3 - 1$

$y = x - 3$

Since $0 \leq \cos^2\theta \leq 1$

$0 \leq 2\cos^2\theta \leq 2$

$\therefore -3 \leq -3 + 2\cos^2\theta \leq -1$

$-3 \leq x \leq -1$

3 Correct solution

$\therefore y = x - 3, \quad -3 \leq x \leq -1$

2 Correct Cartesian equation

1 Eliminates parameter θ by using valid trig. identity.

11e) $2x^3-4x^2+1=0$

$\Sigma\alpha=\frac{4}{2}=2 \qquad \Sigma\alpha\beta=0 \qquad \Pi\alpha=-\frac{1}{2}$

i) $\alpha^2+\beta^2+\gamma^2=(\Sigma\alpha)^2-2(\Sigma\alpha\beta)$

$=\left(\frac{4}{2}\right)^2-2(0)$

$=4$

2 Correct solution

1 Uses $\alpha^2+\beta^2+\gamma^2=(\Sigma\alpha)^2-2(\Sigma\alpha\beta)$

ii) $\frac{\alpha}{\beta\gamma}+\frac{\beta}{\alpha\gamma}+\frac{\gamma}{\alpha\beta}$

$\frac{\alpha^2+\beta^2+\gamma^2}{\alpha\beta\gamma}=\frac{(\Sigma\alpha)^2-2(\Sigma\alpha\beta)}{\alpha\beta\gamma}$

$=\frac{(2)^2-0}{-1/2}$

$=\frac{4}{-1/2}=-8$

1 Correct solution

11f)

i) $f(x)=2\cos^{-1}x$

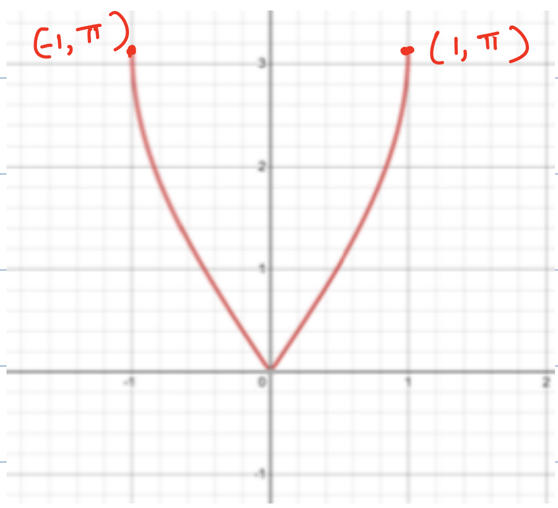
Domain: $\{x: -1\le x\le 1\}$

Range: $\{y: 0\le y\le 2\pi\}$

2 marks Correct Solution

1 mark Correct domain or range of $f(x)$.

ii)



2 marks Correct sketch

1 mark Correct sketch for $2\cos^{-1}x$

or Correct sketch for $2\cos^{-1}x-\pi$

or Correct sketch for $|2\cos^{-1}x|$

Note: Sketch should have correct shape including endpoints and cannot look "smooth" at the origin.

12 (a)

i) $P(-1) = (-1)^3 - (k-1)(-1)^2 + (1-k)(-1) + 1$

$= -1 - (k-1) - (1-k) + 1$ — 1 mark Correct Solution.

$= -1 - k + 1 - 1 + k + 1$ — shows $P(1)=0$ by substitution.

$= 0$

ii) Since $P(x)=0$ has 3 real roots — 3 marks Correct Solution

$\therefore\ x^2 - kx + 1 = 0$ has 2 real roots — 2 marks Finds discriminant

$\therefore\ \Delta > 0$ — of quadratic in terms of k

$k^2 - 4 > 0$ — and realise $\Delta > 0$ for real roots

$(k-2)(k+2) > 0$ — 1 mark Deduce $x^2+kx+1=0$

$\therefore\ k < -2$ or $k > 2$. — has real roots.

12 b)

i) $T = T_0 + Ae^{kt}$ —— (1) — 1 mark correct solution.

$\frac{dT}{dt} = \frac{d}{dt}\left(T_0 + Ae^{kt}\right)$

$= kAe^{kt}$

$= k(T - T_0)$ [from (1) $T - T_0 = Ae^{kt}$]

$\therefore$ (1) satisfies the given differential equation.

ii) At $t=0$, $T=96$, $T_0=18$

$\therefore\ 96 = 18 + A \Rightarrow A = 78$

$\therefore\ T = 18 + 78e^{kt}$

When $t=3$, $T=75^\circ$

$\therefore\ 75 = 18 + 78e^{3k}$

$$e^{3k} = \frac{57}{78}$$

$$3k = \ln\frac{57}{78}$$

$$k = -0.1046$$

2 Correct solution.

1 find A or

Sub $t=3$, $T=75^\circ$ in

$T = T_0 + Ae^{kt}$

iii) $t=?$ $T=60^\circ$

$$T = 18 + 78e^{kt}$$

$$60 = 18 + 78e^{kt}$$

$$e^{kt} = \frac{42}{78}$$

$$kt = \ln\frac{42}{78}$$

$$t = \frac{1}{k}\ln\frac{42}{78} \quad \text{where } k = -0.1046$$

≈ 5.92 minutes.

6 minutes

2 correct solution

1 Sub $T=60^\circ C$ in

$T = T_0 + Ae^{kt}$ to find e^{kt}.

12 c)

i) There are 4 players numbered '6' and 4 players numbered '8' from a total of forty players.

Three '6's can be selected in $^4C_3$ ways

Two '8's can be selected in $^4C_2$ ways

Five players can be selected in $^{40}C_5$ ways.

$\therefore$ Probability $= \dfrac{^4C_3 \times {^4C_2}}{^{40}C_5}$

$= \dfrac{4 \times 6}{658\,008}$

$= \dfrac{1}{27\,417}$

2 correct solution.

1 Correctly find total $^{40}C_5$ (or) $^4C_3 \times {^4C_2}$ (or) equivalent approach.

ii) 'At least 4 players' means 4 or 5 players.

5 players from one team can be selected in $^{10}C_5$ ways.

Since there are 4 teams, therefore 5 players from the same team can be selected in $(^4C_1 \times {^{10}C_5})$ ways

4 players from one team and one player from the remaining teams (30 players) can be selected in $(^4C_1 \times {^{10}C_4} \times {^{30}C_1})$

$\therefore$ Probability $= \dfrac{(^4C_1 \times {^{10}C_5}) + (^4C_1 \times {^{10}C_4} \times {^{30}C_1})}{^{40}C_5}$

$= \dfrac{28}{703}$

2 Correct Solution.

1 Correctly find $^4C_1 \times {^{10}C_5}$ (or) $^4C_1 \times {^{10}C_4} \times {^{30}C_1}$ (or) equivalent approach

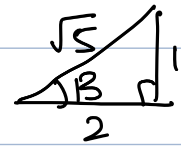
12 d)

Prove $\tan^{-1}\frac{2}{3} + \cos^{-1}\frac{2}{\sqrt{5}} = \tan^{-1}\frac{7}{4}$

or $\tan\left(\tan^{-1}\frac{2}{3} + \cos^{-1}\frac{2}{\sqrt{5}}\right) = \frac{7}{4}$

Let $\tan^{-1}\frac{2}{3} = A$ and $\cos^{-1}\frac{2}{\sqrt{5}} = B$

$\tan A = \frac{2}{3}$ $\quad$ $\cos B = \frac{2}{\sqrt{5}}$



$\tan B = \frac{1}{2}$

$$L.H.S = \tan(A+B)$$

$$= \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{2}{3} + \frac{1}{2}}{1 - \frac{2}{3} \times \frac{1}{2}}$$

$$= \frac{7/6}{4/6}$$

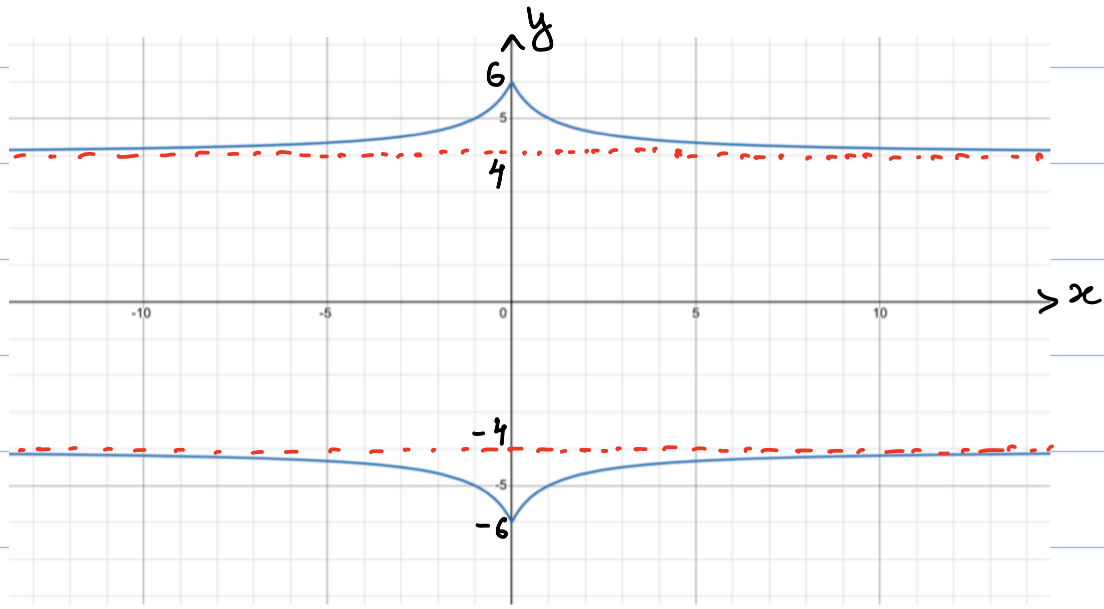
$$= \frac{7}{4}$$

$$= R.H.S$$

2 Correct proof

1 Obtaining $\tan B = \frac{1}{2}$ or equivalent approach.

13 a) i)

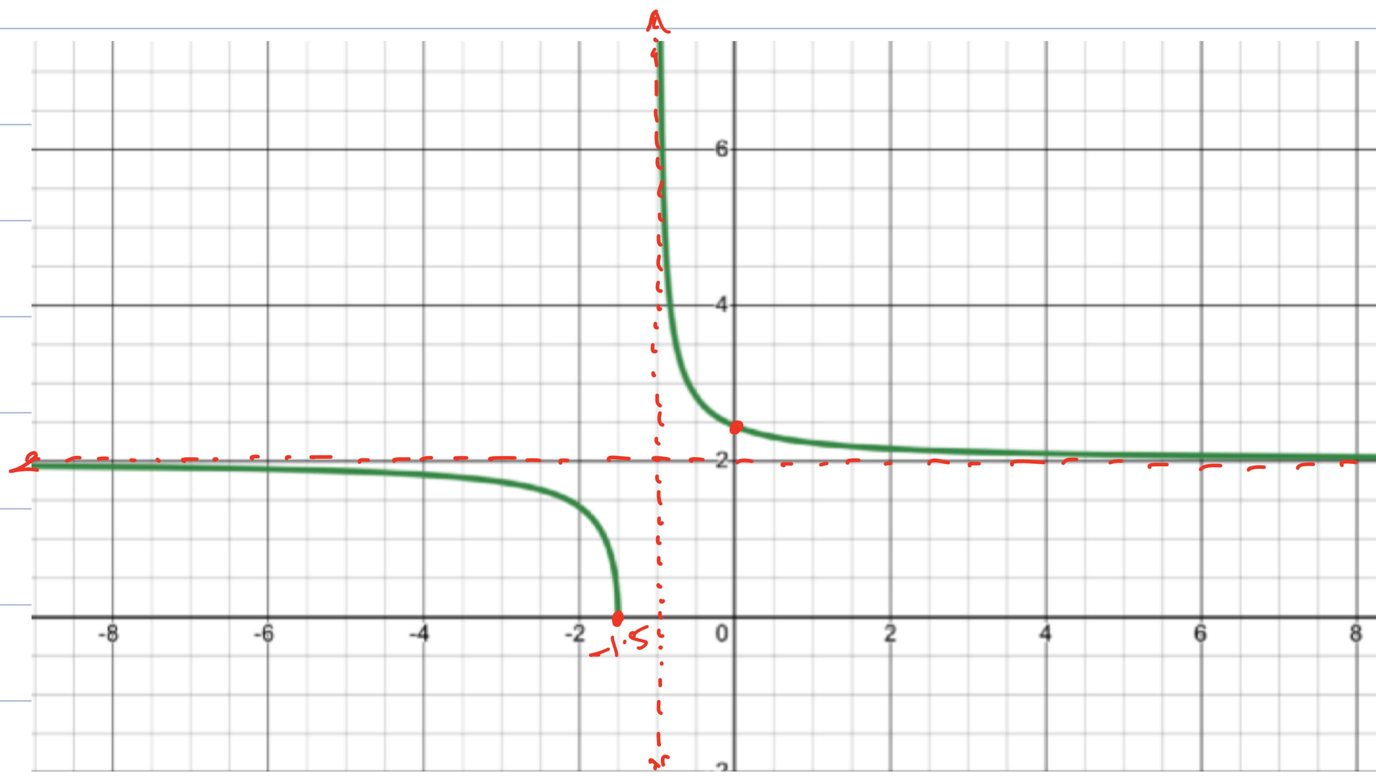


2 marks Correct sketch including shape and asymptotes.

1 mark One correct graph including shape & asymptotes

or both correct graph ($|y|$ & $f(|x|)$) without asymptotes

ii)



2 mark Correct graph including asymptotes.

1 mark Correct graph without asymptotes

(or) Correct graph for any one branch (either $x > -1$ or $x < -1$) including shape and x-intercept.

13 b)

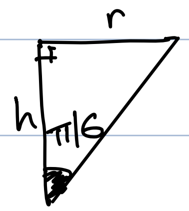
i) $V = \frac{1}{3}\pi r^2 h$

$r = h\tan\frac{\pi}{6}$

$= \frac{h}{\sqrt{3}}$

$\therefore V = \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$

$= \frac{1}{9}\pi h^3$



1 mark Correct solution.

Express radius of the cone in terms of h and show substitution.

ii) $\frac{dh}{dt} = -0.05$ cm/s , $h = ?$

$\frac{dV}{dt} = -0.5$ cm³/s

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$-0.5 = \frac{1}{3}\pi h^2 \times (-0.05)$

$h^2 = \frac{30}{\pi}$

$h = 3.09$ cm

3 marks Correct solution.

2 marks Express the chain rule in terms of h.

1 mark Obtains relevant chain rule 'or' finds $\frac{dV}{dh}$

13c) $\sin x + \cos x = k$

Square both sides

$(\sin x + \cos x)^2 = k^2$

$\sin^2 x + \cos^2 x + 2\sin x\cos x = k^2$

$1 + 2\sin x\cos x = k^2$ $\quad (\because \sin^2 x + \cos^2 x = 1)$

$2\sin x\cos x = k^2 - 1$ 2 Correct Solution

$\sin 2x = k^2 - 1$ 1 Square both sides for

$\sin x + \cos x = k$ (or)

correct use of trig. Identities

13d) $f(x) = ax^3 + bx^2 + cx + d.$

i) $f'(x) = 3ax^2 + 2bx + C$ 1 mark Correct solution

ii) If the inverse of $f(x)$ is not a function, then it is neither monotonically increasing / decreasing.

Since $f'(x)$ is a quadratic function and we don't want $f'(x) \geq 0$ or $f'(x) \leq 0$ for all value of x.

Therefore for $f'(x)$, Set $\Delta > 0$.

$f'(x) = 3ax^2 + 2bx + C$ 2 marks Correct Solution

$\Delta = (2b)^2 - 4(3a)(c) > 0$ 1 mark finds discriminant

$4b^2 - 12ac > 0$ for $f'(x)$ 'or'

or $b^2 - 3ac > 0$ Explains why $\Delta > 0$ for

Hence proved. $f'(x)$.

iii) Compare the cofficient of $g(x)$ and $f(x)$.

$a = \frac{1}{2}$, $b = -3$, $c = 6$, $d = -8$.

For $g^{-1}(x)$ to exist; $b^2 - 3ac = 0$, which shows only one x- intercept.

$$\therefore \; b^2 - 3ac = (-3)^2 - 3\left(\tfrac{1}{2}\right)(6)$$

$$= 9 - 9$$

$$= 0$$

Since $b^2 - 3ac = 0$, therefore from ii) inverse of $g(x)$ is a function.

2 marks Correct solution

1 mark Evaluate $b^2 - 3ac$

14 a)

$$|x-1| > 2\sqrt{x(1-x)}$$

$\sqrt{x(1-x)}$ is defined for $0 \leq x \leq 1$

$$|x-1|^2 > \left(2\sqrt{x(1-x)}\right)^2$$

$$(x-1)^2 > 4x(1-x)$$

$$x^2 - 2x + 1 > 4x - 4x^2$$

$$5x^2 - 6x + 1 > 0$$

$$5x^2 - 5x - x + 1 > 0$$

$$5x(x-1) - (x-1) > 0$$

$$(5x-1)(x-1) > 0$$

$$\therefore \; x < \frac{1}{5} \quad \text{or} \quad x > 1$$

Since function is defined for $0 \leq x \leq 1$

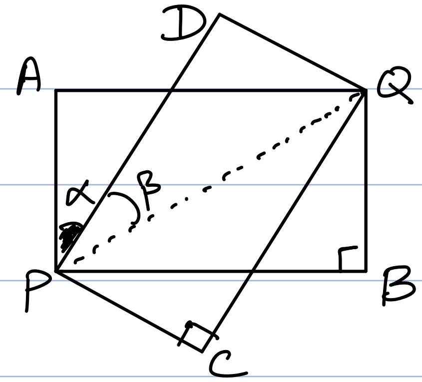
$$\therefore \; 0 \leq x < \frac{1}{5}$$

3 marks - correct solution (graphical / algebraic)

2 marks - Correctly solve the quadratic inequality.

1 mark - Identify the function is defined as $0 \leq x \leq 1$, or square both sides to find correct quad. eq.

14b)



3 correct solution

2 Use Compound angle formula for $\tan(\alpha+\beta)$ (or) equivalent approach

1 Find $\tan\beta = \frac{1}{x}$ or correct approach to find relation between angles

Let $\angle QPD = \beta$

In $\triangle PDQ$; $\tan\beta = \frac{1}{x}$

In $\triangle APQ$; $\tan(\alpha+\beta) = x$

$$\therefore \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = x$$

$$\frac{\tan\frac{\pi}{3} + \frac{1}{x}}{1 - \tan\frac{\pi}{3}\cdot\frac{1}{x}} = x \quad \left(\because \alpha = \frac{\pi}{3} \ \& \ \tan\beta = \frac{1}{x}\right)$$

$$\frac{\sqrt{3} + \frac{1}{x}}{1 - \frac{\sqrt{3}}{x}} = x$$

$$\frac{\sqrt{3}x + 1}{x - \sqrt{3}} = x$$

$$\sqrt{3}x + 1 = x^2 - \sqrt{3}x$$

$$x^2 - 2\sqrt{3}x - 1 = 0$$

$$x = \frac{2\sqrt{3} \pm \sqrt{12+4}}{2}$$

$$= \frac{2\sqrt{3} \pm 4}{2}$$

$$= \sqrt{3} \pm 2$$

$$= \sqrt{3} + 2 \quad (\because x > 0)$$

Hence Shown

14c) L.H.S $= x^n (1+x)^n \left(1+\frac{1}{x}\right)^n$

$= x^n (1+x)^n \left(\frac{x+1}{x}\right)^n$

$= x^n (1+x)^n \frac{(x+1)^n}{x^n}$

$= (1+x)^n (x+1)^n$

$= \left[(1+x)^n\right]^2$

$= (1+x)^{2n}$

$=$ R.H.S

Hence Shown.

2 Correct proof

1 Express $\left(1+\frac{1}{x}\right)^n$ as $\left(\frac{x+1}{x}\right)^n$

'or' $x^n\left(1+\frac{1}{x}\right)^n = (1+x)^n$

(or) equivalent approach.

ii) Compare the coefficient of x^n for part i)

Coefficient of x^n in the expansion of $(1+x)^{2n}$ is ${}^{2n}C_n$.

$$L.H.S = x^n(1+x)^n\left(1+\frac{1}{x}\right)^n$$

$$= x^n\left(1+{}^nC_1x+{}^nC_2x^2+\dots+{}^nC_nx^n\right)\left[1+{}^nC_1\left(\frac{1}{x}\right)+{}^nC_2\left(\frac{1}{x}\right)^2+\dots+{}^nC_n\left(\frac{1}{x}\right)^n\right]$$

for the coefficient of x^n in this expansion

$$x^n\,(\text{constt. term})\,(\text{constt. term}) = x^n.1.1 = 1$$

$$x^n \cdot {}^nC_1x \cdot {}^nC_1\left(\frac{1}{x}\right) = {}^nC_1 \cdot {}^nC_1 = ({}^nC_1)^2$$

$$x^n \cdot {}^nC_2x^2 \cdot {}^nC_2\left(\frac{1}{x}\right)^2 = {}^nC_2 \cdot {}^nC_2 = ({}^nC_2)^2$$

$$\vdots$$

$$x^n \cdot {}^nC_nx^n \cdot {}^nC_n\left(\frac{1}{x}\right)^n = {}^nC_n \cdot {}^nC_n = ({}^nC_n)^2$$

$\therefore$ Coefficient of x^n in the expansion of L.H.S is

$$1+({}^nC_1)^2+({}^nC_2)^2+\dots+({}^nC_n)^2$$

By comparing the coefficient of x^n of both sides of part i)

we get;

$$1+({}^nC_1)^2+({}^nC_2)^2+\dots+({}^nC_n)^2 = {}^{2n}C_n$$

Hence shown.

3 correct proof

2 Correct expansion of coefficient of x^n for $x^n(1+x)^n\left(1+\frac{1}{x}\right)^n$, and $(1+x)^{2n}$ by using binomial expansion

1 Correct expansion of $(1+x)^{2n}$ by using binomial expansion or finding the coefficient of x^n for $(1+x)^{2n}$

14 d) $\dfrac{\sin\beta}{\sin(2\alpha+\beta)} = \dfrac{m}{n}$ ——— (1)

Prove that $\dfrac{n-m}{n+m}\tan(\alpha+\beta) = \tan\alpha$

$$R.H.S = \frac{n-m}{n+m}\tan(\alpha+\beta)$$

$$= \frac{1-\frac{m}{n}}{1+\frac{m}{n}}\tan(\alpha+\beta)$$

$$= \frac{1-\frac{\sin\beta}{\sin(2\alpha+\beta)}}{1+\frac{\sin\beta}{\sin(2\alpha+\beta)}}\tan(\alpha+\beta) \text{ ——— (using (1))}$$

$$= \frac{\sin(2\alpha+\beta)-\sin\beta}{\sin(2\alpha+\beta)+\sin\beta}\cdot\tan(\alpha+\beta)$$

$$= \frac{2\cos\left(\frac{2\alpha+\beta+\beta}{2}\right)\sin\left(\frac{2\alpha+\beta-\beta}{2}\right)}{2\sin\left(\frac{2\alpha+\beta+\beta}{2}\right)\cos\left(\frac{2\alpha+\beta-\beta}{2}\right)}\cdot\tan(\alpha+\beta)$$

$$= \frac{\cos(\alpha+\beta)\sin\alpha}{\sin(\alpha+\beta)\cos\alpha}\cdot\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$$

$$= \frac{\sin\alpha}{\cos\alpha}$$

$$= \tan\alpha$$

$$= L.H.S$$

3 correct proof.

2 Simplify $\sin(2\alpha+\beta)$ and $\tan(\alpha+\beta)$ by using correct trig. Identity (OR) use of product to sum results.

1 Eliminating m and n.

END OF EXAM

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