Baulkham Hills High School

2022 **YEAR 11 YEARLY EXAMINATION**

Mathematics Extension

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 11)

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

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1 What is the domain and range of $y = \frac{1}{2}\cos^{-1}\left(\frac{x}{2}\right)$?

A. Domain: $-\frac{1}{2} \le x \le \frac{1}{2}$, range: $0 \le y \le \frac{\pi}{2}$.

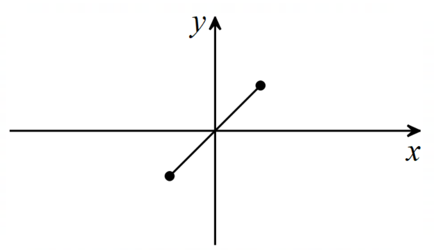
B. Domain: $-\frac{1}{2} \le x \le \frac{1}{2}$, range: $0 \le y \le 2\pi$.

C. Domain: $-2 \le x \le 2$, range: $0 \le y \le \frac{\pi}{2}$.

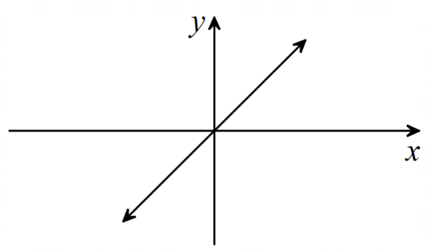
D. Domain: $-2 \le x \le 2$, range: $0 \le y \le 2\pi$.

2 Which of the following is the best representation of the graph of $y = \arcsin(\sin x)$?

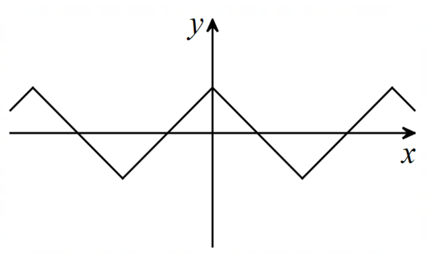
A.



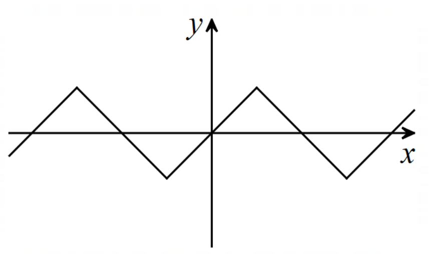
B.



C.



D.



3 In a jar, there are many identical marbles only different in colour. Kevin wants to get 4 marbles that match in colour by reaching into the jar and pulling marbles out at random, one at a time until he has four of matching colour. It is known that Kevin needs to pull out a minimum of 13 marbles to guarantee this. How many different colours of marbles are there in the jar?

A. 3

B. 4

C. 12

D. 14

4 What type of relation is $x|y|=1$?

A. One-to-one

B. One-to-many

C. Many-to-one

D. Many-to-many

5 What is the value of $\cos^{-1}\left(\cos\frac{6\pi}{5}\right)$?

A. $\frac{6\pi}{5}$

B. $\frac{4\pi}{5}$

C. $\frac{\pi}{5}$

D. $-\frac{\pi}{5}$

6 Which of the following expressions is equivalent to $\cos 2x$ for all values of x?

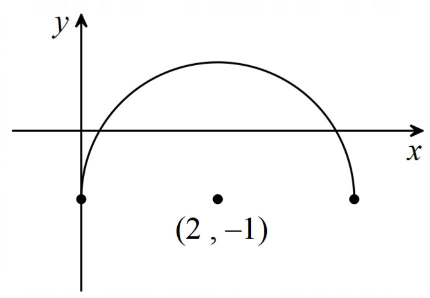
A. $\sin^2 x - \cos^2 x$

B. $\cos 3x \cos x - \sin 3x \sin x$

C. $\dfrac{1-\tan^2 x}{1+\tan^2 x}$

D. $1-\sin^2 2x$

7 The diagram below shows a semicircular arc of radius 2, centred at $(2,-1)$.



Which of the following pair of parametric equations represents this arc?

A. $x = 2\cos\theta + 2$, $y = 2\sin\theta - 1$ from $\theta = 0$ to $\theta = \pi$.

B. $x = 2\sin\theta + 2$, $y = 2\cos\theta - 1$ from $\theta = 0$ to $\theta = \pi$.

C. $x = 2\cos\theta + 2$, $y = 2\sin\theta + 1$ from $\theta = 0$ to $\theta = \pi$.

D. $x = 2\sin\theta + 2$, $y = 2\cos\theta + 1$ from $\theta = 0$ to $\theta = \pi$.

8 Consider the polynomial $P(x) = (k^3 - k)x^4 + (k^2 - k)x^3 + x^2$, what value(s) of k will make $P(x)$ a non-zero polynomial of degree 3?

A. $k = -1$

B. $k = -1,\ k = 0$

C. $k = -1,\ k = 1$

D. $k = -1,\ k = 0,\ k = 1$

9 Given that $P(x)$ is a polynomial of a non-zero even degree and $Q(x)$ is a polynomial of odd degree, neither of which are zero polynomials, which of the following statements is **<u>ALWAYS TRUE</u>**?

A. $P(x)+Q(x)$ is of a non-zero even degree.

B. $P(x)-Q(x)$ is of a non-zero even degree.

C. $P(x)Q(x)$ is of a non-zero even degree.

D. $P(Q(x))$ is of a non-zero even degree.

10 The function $f(x)$ is strictly increasing with domain $a \le x \le c$ and range $b \le y \le d$, where *a*, *b*, *c* and *d* are constants. Let $g(x)$ be the inverse of $f(x)$, which of the following statements is **<u>ALWAYS TRUE</u>**?

A. $f'(a)=g'(b)$

B. $f'(a)<f'(c)$

C. $g'(b)<g'(d)$

D. $f(g(b))<f(g(d))$

End of Section I

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question on the appropriate pages of the writing booklet. Extra writing paper is available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 section of the writing booklet.

(a) Solve $|2x-3|<5$. **2**

(b) Solve $\dfrac{x+5}{x+3} \geq \dfrac{1}{2}$. **3**

(c) A rock drops into a pond creating a circular ripple. The radius of the ripple increases from 0 cm, at a constant rate of 8 centimetres per second. At what rate is the area enclosed within the ripple increasing when the radius is 14 cm? **2**

(d) A committee of five is to be formed from a group of 5 men and 9 women. In how many ways can this be done if:

(i) there are no restrictions? **1**

(ii) the committee consists of a majority of women? **2**

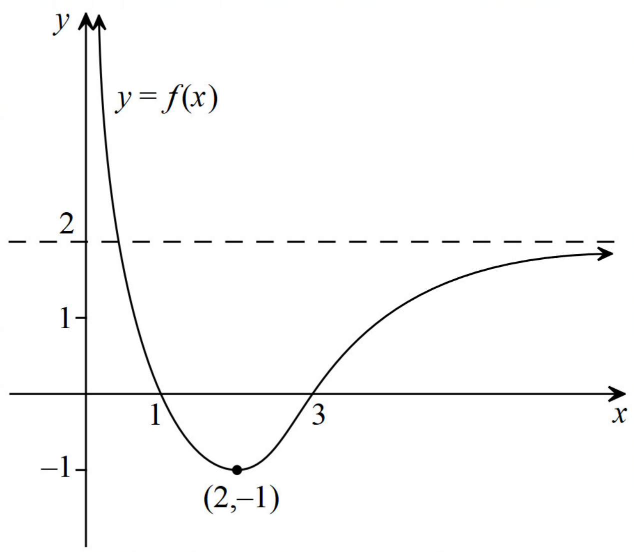
(iii) two of the men refuse to serve on the same committee? **2**

(e) Using $t=\tan\dfrac{x}{2}$, prove that $\dfrac{1+\sin x+\cos x}{1-\sin x+\cos x}=\tan x+\sec x$. **3**

End of Question 11

Question 12 (15 marks) Use the Question 12 section of the writing booklet.

(a) Consider the graph of $y = f\left(x\right)$ below.



On separate axes, sketch each of the following, showing all important features.

(i) $y^2 = f\left(x\right)$ **3**

(ii) $y = \dfrac{1}{f\left(\left|x\right|\right)}$ **3**

(b) Find, as a rational number, the coefficient of x in the expansion of $\left(x^2 + \dfrac{1}{2x}\right)^8$. **3**

Question 12 continues on the next page

Question 12 (continued)

(c) A bottle of water has a temperature of $22°\text{C}$ and is placed in a refrigerator in which the temperature is $4°\text{C}$. The cooling rate of the bottle of water is proportional to the difference between the temperature of the refrigerator and the temperature T of the bottle of water in degrees Celsius. This is modelled by the equation

$$\frac{dT}{dt} = -k(T-4)$$

where k is a constant of proportionality and t is the number of minutes after the bottle of water is placed in the refrigerator.

(i) Show that $T = 4 + Ae^{-kt}$ satisfies the above equation. **1**

(ii) After 25 minutes in the refrigerator, the temperature of the bottle of water is $8°\text{C}$. Find the values of A and k in the equation in part (i). **3**

(iii) How long will it take for the bottle of water to cool down to $6°\text{C}$? Express your answer correct to the nearest minute. **2**

End of Question 12

Question 13 (15 marks) Use the Question 13 section of the writing booklet.

(a) (i) Prove the identity $\tan\left(\phi-\frac{\pi}{4}\right)=\frac{\sin 2\phi-1}{\cos 2\phi}$. **3**

(ii) Hence evaluate $\tan\frac{\pi}{12}$, express your answer as an exact value. **1**

(b) Consider the polynomial $f(x)=px^3-4x^2-11x+q$ where p and q are constants. Given that $(x-2)$ and $(2x+3)$ are both factors of $f(x)$, find the values of p and q, and hence fully factorise $f(x)$. **4**

(c) The inverse of the function given by the equation $f(x)=2-\sqrt{x+5}$ is $f^{-1}(x)$. State the domain and range of $f^{-1}(x)$. **2**

(d) The letters of the word **ELLIPTICAL** are arranged in a line.

(i) How many different arrangements are possible if there are no restrictions? **2**

(ii) How many different arrangements are possible if no **L**'s are adjacent to another **L**? **2**

(iii) Treating each possible arrangement in part (i) as a word, if one such word is chosen at random, what is the probability that the word has the letters **C**, **A**, **P**, **E** in that order? As examples: **1**

Words the match the description	Words that do NOT match the description
CLLATPIEIL	**PLLATCIEIL**
LLCATIPIEL	**LLEPTICIAL**
CLTIAPELLI	**ELTIPACLLI**
LITICAPELL	**LITICPAELL**

End of Question 13

Question 14 (15 marks) Use the Question 14 section of the writing booklet.

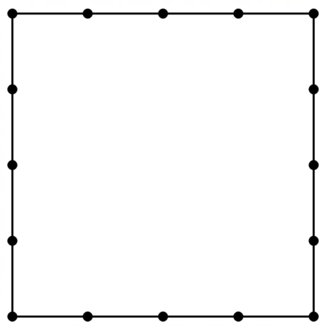
(a) The roots of the polynomial $x^3 - x^2 - 5x + 2 = 0$ are α , β and γ .

(i) Write down the values of $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **2**

(ii) Show that $\beta + \gamma = 1 - \alpha$. **1**

(iii) Hence evaluate $\dfrac{\alpha+\beta}{\gamma} + \dfrac{\beta+\gamma}{\alpha} + \dfrac{\gamma+\alpha}{\beta}$. **2**

(b) Sixteen points are equally spaced along the perimeter of a square as shown below. **2**



How many triangles can be formed if the vertices of the triangles are to be chosen from these sixteen points?

(c) By considering the expansion of $(1+x)^n$ and differentiating, show the following identity given n is even. **4**

$$n2^{n-2} = \binom{n}{1} + 3\binom{n}{3} + 5\binom{n}{5} + \cdots + (n-1)\binom{n}{n-1}$$

Question 14 (continued)

(d) The function $y = P(x)$ is defined by $P(x) = (x - p)(x - q)(x - r)$, where p, q and r are distinct real numbers.

(i) Sketch a possible graph of $y = P(x)$ without using calculus. **1**

(ii) Expand $P(x)$, expressing it in the form $x^3 + bx^2 + cx + d$. **1**

(iii) The points on $P(x)$ at which the gradient of the tangent is zero can be found by solving the equation $P'(x) = 0$. Consider the graph of $y = P(x)$, without solving the equation $P'(x) = 0$, prove that $(p+q+r)^2 > 3(pq+qr+rp)$. **2**

End of Paper

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YEAR 11 YEARLY EXAMINATION 2022
MATHEMATICS EXTENSION
MARKING GUIDELINES

Section I

Multiple-choice Answer Key

Question	Answer
1	C
2	D
3	B
4	B
5	B

Question	Answer
6	C
7	A
8	A
9	D
10	D

Questions 1 – 10

Sample solution

1. Domain:

$$-1 \le \frac{x}{2} \le 1$$
$$-2 \le x \le 2$$

Range:

$$0 \le \cos^{-1}\left(\frac{x}{2}\right) \le \pi$$
$$0 \le \frac{1}{2}\cos^{-1}\left(\frac{x}{2}\right) \le \frac{\pi}{2}$$
$$0 \le y \le \frac{\pi}{2}$$

2. Domain of $y = \arcsin(\sin x)$ is all real x.

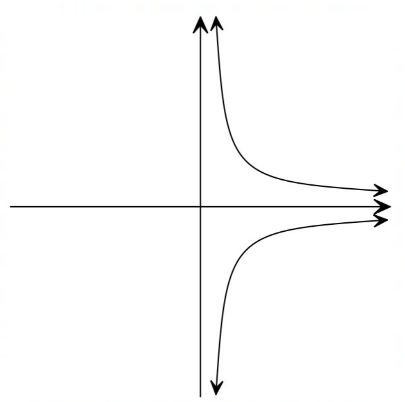
Also,

$$\arcsin(\sin 0) = \arcsin 0$$
$$= 0$$

3. With 3 colours, worse case scenario is 3 of each colour, totalling to 9 marbles, the tenth marble will then guarantee that Kevin has four marbles of the same colour.

With 4 colours, worse case scenario is 3 of each colour, totalling to 12 marbles, the thirteenth marble will then guarantee that Kevin has four marbles of the same colour.

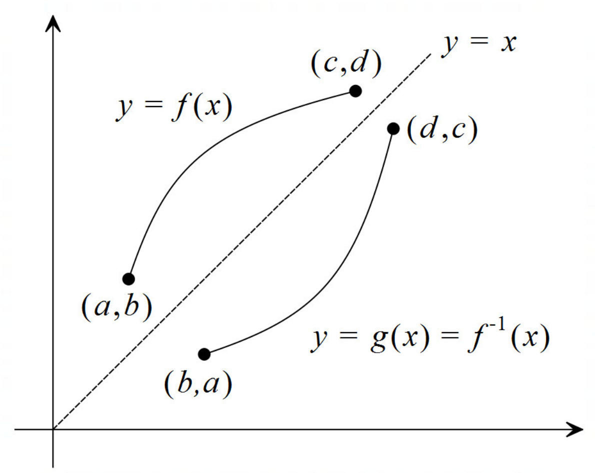
4. The graph of $x|y| = 1$ is:



This is a one-to-many relation.

5.

$$\cos^{-1}\left(\cos\frac{6\pi}{5}\right) = \cos^{-1}\left(\cos\frac{4\pi}{5}\right)$$
$$= \frac{4\pi}{5}$$

<table>
<tr><th colspan="3">Sample solution</th></tr>
<tr><td>6.</td><td>By t-formula where $t=\tan x$,
$$\frac{1-\tan^2 x}{1+\tan^2 x}=\frac{1-t^2}{1+t^2}$$
$$=\cos 2x$$</td><td>Option A simplifies to $-\cos 2x$
Option B:
$$\cos 3x\cos x-\sin 3x\sin x=\cos(3x+x)$$
$$=\cos 4x$$
Option D simplifies to $\cos^2 2x$</td></tr>
<tr><td>7.</td><td colspan="2">A circle of radius 2, centred at the origin, tracing anticlockwise from $(2,0)$ would have parametric equations $x=2\cos\theta$, $y=2\sin\theta$. Translating the centre of this circle to $(2,-1)$ would yield $x-2=2\cos\theta$, $y+1=2\sin\theta$. Rearranging gives $x=2\cos\theta+2$, $y=2\sin\theta-1$.</td></tr>
<tr><td>8.</td><td colspan="2">For $P(x)$ to be of degree 3, $k^3-k=0$, but $k^2-k\neq 0$.
$$k^3-k=0$$
$$k(k^2-1)=0$$
$$k(k+1)(k-1)=0$$
$$k=0,\pm 1$$
$$k^2-k=0$$
$$k(k-1)=0$$
$$k(k-1)=0$$
$$k=0,1$$
Therefore, the only value that will make $P(x)$ a polynomial of degree 3 is when $k=-1$.</td></tr>
<tr><td>9.</td><td colspan="2">Option A not always true, false when $\deg(Q(x))>\deg(P(x))$.
Option B not always true, false when $\deg(P(x))<\deg(Q(x))$.
Option C not true at all as an even degreed polynomial multiplied by an odd degreed polynomial gives an odd degreed polynomial.
Option D is true, let m be the degree of $P(x)$ and n be the degree of $Q(x)$. Consider only the leading term of each polynomial as this will dictate the leading term of $P(Q(x))$. We can ignore the associated coefficients as they are non-zero by definition and negligible when determining the degree of $P(Q(x))$. The leading term would be $(x^n)^m=x^{mn}$.
Since m is even and n is odd, mn is even.</td></tr>
<tr><td>10.</td><td>
</td><td>Clearly, the gradients at $x=a$ on $f(x)$ and at $x=b$ on $g(x)$ are not necessarily equal.
In this example, $f'(a)\nless f'(c)$, but in the case where $f(x)$ is strictly increasing but concave up, it is possible for $f'(a)<f'(c)$.
If $f(x)$ is increasing, then $g(x)$ would be too, using a similar argument as option B, $g'(b)<g'(d)$ is not always true.
Option D is always true, as $f(g(b))=f(a)=b$ and $f(g(d))=f(c)=d$. We know $b<d$ because $f(x)$ is an increasing function.</td></tr>
</table>

Section II

Question 11

Sample solution		Suggested marking criteria
(a)	$\lvert 2x-3 \rvert < 5$ $-5 < 2x-3 < 5$ $-2 < 2x < 8$ $-1 < x < 4$	• 2 – correct solution • 1 – solves $2x-3<5$ or equivalent merit
(b)	$\dfrac{x+5}{x+3} \geq \dfrac{1}{2}$ $\dfrac{2x+10}{x+3} \geq 1$ $\dfrac{2x+10}{x+3} - 1 \geq 0$ $\dfrac{2x+10-x-3}{x+3} \geq 0$ $\dfrac{x+7}{x+3} \geq 0$ $(x+7)(x+3) \geq 0$ $\therefore x \leq -7, x > -3 \quad (\text{since } x \neq -3)$	• 3 – correct solutions • 2 – solves the inequation using an appropriate method but includes $x=-3$ • 1 – uses an appropriate method to attempt to solve the inequation
(c)	$\dfrac{dr}{dt} = 8$ $A = \pi r^2$ $\dfrac{dA}{dr} = 2\pi r$ $\dfrac{dA}{dt} = \dfrac{dA}{dr} \times \dfrac{dr}{dt}$ $= 2\pi r \times 8$ $= 16\pi r$ When $r = 14$, $\dfrac{dA}{dt} = 16\pi \times 14$ $= 224\pi$ $= 703.72 \text{ cm}^2/\text{s} \quad (2 \text{ d.p.})$	• 2 – correct solution • 1 – correct expression for $\dfrac{dA}{dt}$
(d)	(i) $^{14}C_5 = 2002$ *(Note: Accept answers expressed in either form.)*	• 1 – correct answer
	(ii) $^9C_3 \times {^5C_2} + {^9C_4} \times {^5C_1} + {^9C_5} = 1596$	• 2 – correct solution • 1 – correctly calculates a subset of all possible cases
	(iii) Number of committees with neither men $= {^{12}C_5}$ $= 792$ Number of committees with one of the two men $= 2 \times {^{12}C_4}$ $= 990$ Total number of possible committees $= 1782$	• 2 – correct solutions • 1 – correctly calculates a subset of all possible cases

Question 11 (continued)

<table>
<tr><th colspan="3">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(e)</td><td>

$$\frac{1+\sin x+\cos x}{1-\sin x+\cos x}=\frac{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}{1-\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$$
$$=\frac{1+t^2+2t+1-t^2}{1+t^2-2t+1-t^2}$$
$$=\frac{2+2t}{2-2t}$$
$$=\frac{1+t}{1-t}$$
$$=\frac{(1+t)^2}{(1-t)(1+t)}$$
$$=\frac{1+2t+t^2}{1-t^2}$$
$$=\frac{2t}{1-t^2}+\frac{1+t^2}{1-t^2}$$
$$=\frac{2t}{1-t^2}+\frac{1}{\left(\frac{1-t^2}{1+t^2}\right)}$$
$$=\tan x+\frac{1}{\cos x}$$
$$=\tan x+\sec x$$

</td><td>

Alternatively,

$$\frac{1+\sin x+\cos x}{1-\sin x+\cos x}=\frac{1+\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}{1-\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$$
$$=\frac{1+t^2+2t+1-t^2}{1+t^2-2t+1-t^2}$$
$$=\frac{2+2t}{2-2t}$$
$$=\frac{1+t}{1-t}$$

$$\tan x+\sec x=\tan x+\frac{1}{\cos x}$$
$$=\frac{2t}{1-t^2}+\frac{1+t^2}{1-t^2}$$
$$=\frac{1+2t+t^2}{1-t^2}$$
$$=\frac{(1+t)^2}{(1-t)(1+t)}$$
$$=\frac{1+t}{1-t}$$

$$\frac{1+\sin x+\cos x}{1-\sin x+\cos x}=\frac{1+t}{1-t}=\tan x+\sec x$$

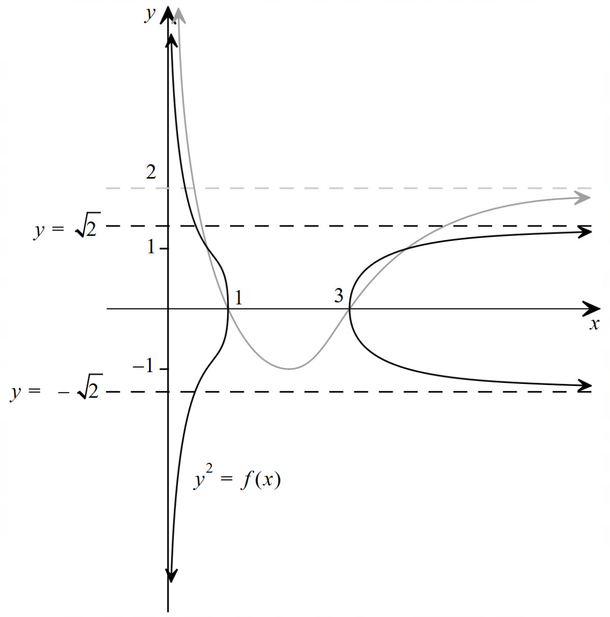
</td><td>

- 3 – correct proof
- 2 – correctly simplifies the expression in terms of t towards a correct proof
- 1 – correctly uses t-formulas to transform the given expression

</td></tr>
</table>

Question 12

<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(a)</td><td>(i)



</td><td>

- 3 – correct graph
- 2 – correctly graphs $y = \sqrt{f(x)}$
- 1 – uses correct graphing techniques to attempt to graph $y = \sqrt{f(x)}$

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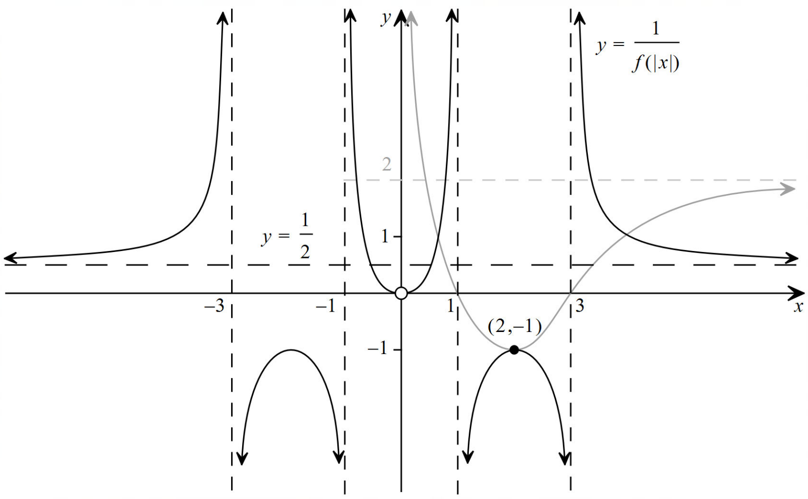
<tr><td></td><td>(ii)



</td><td>

- 3 – correct graph
- 2 – correctly graphs $y = \dfrac{1}{f(x)}$
  - – graphs $y = \dfrac{1}{f(|x|)}$ but with features missing
- 1 – uses correct graphing techniques to attempt to graph $y = \dfrac{1}{f(x)}$
  - – correctly graphs $y = f(|x|)$

</td></tr>
<tr><td>(b)</td><td>

$$\left(x^2 + \frac{1}{2x}\right)^8 = \sum_{k=0}^{8} {}^8C_k \left(x^2\right)^{8-k} \left(\frac{1}{2x}\right)^k$$

$$= \sum_{k=0}^{8} {}^8C_k x^{16-2k} \times \left(\frac{1}{2}\right)^k x^{-k}$$

$$= \sum_{k=0}^{8} {}^8C_k \times \left(\frac{1}{2}\right)^k x^{16-3k}$$

For the coefficient of x:

$$16 - 3k = 1$$
$$3k = 15$$
$$k = 5$$

$$\text{Coefficient of } x = {}^8C_5 \times \left(\frac{1}{2}\right)^5$$
$$= 56 \times \frac{1}{32}$$
$$= \frac{7}{4}$$

</td><td>

- 3 – correct solution
- 2 – correctly finds the value of k
  - – correctly expands the binomial expression and simplifies the terms to include the x term
- 1 – finds the generic term of the expansion
  - – correctly expands the binomial expression

</td></tr>
</table>

Question 12 (continued)

<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(c)</td><td>(i)
$$T = 4 + Ae^{-kt}$$
$$\frac{dT}{dt} = 0 + A \times -ke^{-kt}$$
$$= -kAe^{-kt}$$
$$= -k\left(4 + Ae^{-kt} - 4\right)$$
$$= -k\left(T - 4\right)$$</td><td>• 1 – correctly shows the required result</td></tr>
<tr><td></td><td>(ii)
At $t = 0$, $T = 22$:
$$22 = 4 + Ae^{0}$$
$$18 = A$$
At $t = 25$, $T = 8$:
$$T = 4 + 18e^{-kt}$$
$$8 = 4 + 18e^{-25k}$$
$$4 = 18e^{-25k}$$
$$\frac{2}{9} = e^{-25k}$$
$$-25k = \ln\frac{2}{9}$$
$$k = -\frac{1}{25}\ln\frac{2}{9} \left(\text{or } \frac{1}{25}\ln\frac{9}{2} \approx 0.0602 \ (4 \text{ d.p.})\right)$$</td><td>• 3 – correctly finds the values of A and k
• 2 – correctly states the value of A and uses it in an attempt to find the value of k
• 1 – correct value of A</td></tr>
<tr><td></td><td>(iii)
$$T = 4 + 18e^{-kt}, \text{ where } k = \frac{1}{25}\ln\frac{9}{2}$$
$$6 = 4 + 18e^{-kt}$$
$$2 = 18e^{-kt}$$
$$\frac{1}{9} = e^{-kt}$$
$$-kt = \ln\frac{1}{9}$$
$$-\frac{t}{25}\ln\frac{9}{2} = \ln\frac{1}{9}$$
$$t = -25\frac{\ln\frac{1}{9}}{\ln\frac{9}{2}}$$
$$= 36.5211355\ldots$$
$$= 37 \text{ minutes (nearest minute)}$$</td><td>• 2 – correct solution
• 1 – correctly applies logarithms in an attempt to solve for t</td></tr>
</table>

Question 13

Sample solution		Suggested marking criteria
(a)	(i) $\tan\left(\phi-\frac{\pi}{4}\right)=\frac{\tan\phi-\tan\frac{\pi}{4}}{1+\tan\phi\tan\frac{\pi}{4}}$ $=\frac{\tan\phi-1}{1+\tan\phi}$ $=\frac{\sin\phi-\cos\phi}{\cos\phi+\sin\phi}$ $=\frac{(\sin\phi-\cos\phi)^2}{(\cos\phi+\sin\phi)(\sin\phi-\cos\phi)}$ $=\frac{\sin^2\phi-2\sin\phi\cos\phi+\cos^2\phi}{\sin^2\phi-\cos^2\phi}$ $=\frac{1-\sin 2\phi}{-\cos 2\phi}$ $=\frac{\sin 2\phi-1}{\cos 2\phi}$	• 3 – correct solution • 2 – correctly uses a trigonometric identity in an attempt to convert between ϕ and 2ϕ • 1 – correctly uses the compound angle formula
	(ii) Letting $\phi=\frac{\pi}{3}$: $\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{\sin\frac{2\pi}{3}-1}{\cos\frac{2\pi}{3}}$ $\tan\frac{\pi}{12}=\frac{\frac{\sqrt{3}}{2}-1}{-\frac{1}{2}}$ $=2-\sqrt{3}$	• 1 – correct answer using part (i)
(b)	$f(2)=0$ $p\times 2^3-4\times 2^2-11\times 2+q=0$ $8p-16-22+q=0$ $8p-38+q=0$ $8p+q=38$ ... (1) $f\left(-\frac{3}{2}\right)=0$ $p\times\left(-\frac{3}{2}\right)^3-4\times\left(-\frac{3}{2}\right)^2-11\times\left(-\frac{3}{2}\right)+q=0$ $-\frac{27}{8}p-9+\frac{33}{2}+q=0$ $-27p-72+132+8q=0$ $-27p+60+8q=0$ $-27p+8q=-60$ ... (2) Solving simultaneously, $p=4, q=6$. Let $4x^3-4x-11x+6\equiv(x-2)(2x+3)(2x+k)$, by comparing constant terms, $k=-1$, therefore, $f(x)=(x-2)(2x+3)(2x-1)$.	• 4 – correct solution • 3 – correctly finds the values of p and q • 2 – correctly finds the value of p or q • 1 – correctly uses the factor theorem for $x=2$ or $x=-\frac{3}{2}$

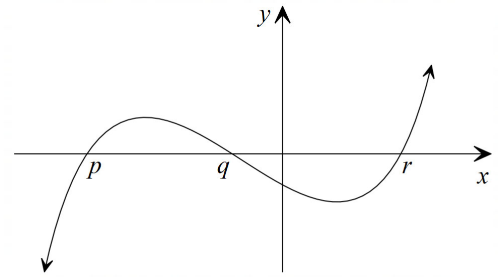
Question 13 (continued)

Sample solution			Suggested marking criteria
(c)		$f(x)$ has a domain of $x \geq -5$ and a range of $y \leq 2$. Therefore $f^{-1}(x)$ has a domain of $x \leq 2$ and a range of $y \geq -5$	• 2 – correct answer • 1 – correct domain for $f^{-1}(x)$ – correct range for $f^{-1}(x)$ – correct domain and range for $f(x)$
(d)	(i)	$\frac{10!}{3! \times 2!} = 302400$	• 2 – correct answer • 1 – uses permutation to arrange the letters of the words
	(ii)	Arrange the letters **E**, **I**, **P**, **T**, **I**, **C**, **A** in $\frac{7!}{2!} = 2520$ ways Then of the 8 spaces (6 spaces between the letters, the space before the first letter and the space after the final letter), choose 3 to slot the **L**'s in, this can be done in ${}^8C_3 = 56$ ways. In total, there are $2520 \times 56 = 141120$ such arrangements.	• 2 – correct answer • 1 – correctly considers some suitable cases, either by totalling or by subtracting the complement from the total
	(iii)	The letters **C**, **A**, **P** and **E** do not repeat and there are $4! = 24$ ways they can be arranged within any word. Therefore, there is a $\frac{1}{24}$ chance of those letters appearing in the given order, no two of which are necessarily together.	• 1 – correct answer

Question 14

Sample solution		Suggested marking criteria
(a)	(i) $\alpha\beta + \beta\gamma + \gamma\alpha = -5$ $\alpha\beta\gamma = -2$	• 2 – correct answers • 1 – correct value for one of the required sum and product of roots
	(ii) $\alpha + \beta + \gamma = 1$ $\beta + \gamma = 1 - \alpha$	• 1 – correct solution
	(iii) $\frac{\alpha+\beta}{\gamma} + \frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} = \frac{1-\gamma}{\gamma} + \frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta}$ $= \frac{1}{\gamma} - 1 + \frac{1}{\alpha} - 1 + \frac{1}{\beta} - 1$ $= \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} - 3$ $= \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} - 3$ $= \frac{-5}{-2} - 3$ $= -\frac{1}{2}$	• 2 – correct solution • 1 – correctly uses a sum or product of roots after appropriate algebraic manipulations
(b)	There are ${}^{16}C_3 = 560$ ways of choosing any 3 points along the perimeter of the square. However, any 3 points chosen along an edge of a square will not form a triangle. There are $4 \times {}^{5}C_3 = 40$ ways this can happen. In total, there are $560 - 40 = 520$ proper triangles that satisfy the given criteria.	• 2 – correct solution • 1 – obtains ${}^{16}C_3$
(c)	$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$ Differentiating gives $n(1+x)^{n-1} = \binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + \cdots + n\binom{n}{n}x^{n-1}$. Setting $x = 1$: $n2^{n-1} = \binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \cdots + n\binom{n}{n}$. Setting $x = -1$: $0 = \binom{n}{1} - 2\binom{n}{2} + 3\binom{n}{3} - \cdots - n\binom{n}{n}$. Adding these identities gives: $n2^{n-1} = 2\binom{n}{1} + 6\binom{n}{3} + 10\binom{n}{5} + \cdots + 2(n-1)\binom{n}{n-1}$ $n2^{n-2} = \binom{n}{1} + 3\binom{n}{3} + 5\binom{n}{5} + \cdots + (n-1)\binom{n}{n-1}$	• 4 – correct proof • 3 – substitutes a suitable value of x in an attempt to derive the binomial identity • 2 – correctly differentiates the expansion of $(1+x)^n$ • 1 – correctly expands $(1+x)^n$

Question 14 (continued)

Sample solution			Suggested marking criteria
(d)	(i)		• 1 – correct graph
	(ii)	$P(x)=(x-p)(x-q)(x-r)$ $=(x-p)(x^2-rx-qx+qr)$ $=x^3-rx^2-qx^2+qrx-px^2+prx+pqx-pqr$ $=x^3-(p+q+r)x^2+(pq+qr+rp)-pqr$	• 1 – correct expansion
	(iii)	$P'(x)=3x^2-2(p+q+r)x+(pq+qr+rp)$ Evidently from the sketch in part (i), there are two points where the gradient of the tangent is zero, as such, $P'(x)=0$ would give two distinct answers. $3x^2-2(p+q+r)x+(pq+qr+rp)=0$ will have two distinct answers when: $\Delta>0$ $\left[-2(p+q+r)\right]^2-4\times 3\times(pq+qr+rp)>0$ $4(p+q+r)^2-12(pq+qr+rp)>0$ $4(p+q+r)^2>12(pq+qr+rp)$ $(p+q+r)^2>3(pq+qr+rp)$	• 2 – correct proof • 1 – finds $P'(x)$