BAULKHAM HILLS HIGH SCHOOL

2020

YEAR 11 YEARLY EXAMINATIONS

Mathematics Extension

General Instructions	• Reading time – 10 minutes • Working time – 2 hours • Write using black pen. • Board-approved calculators may be used • A reference sheet is provided at the back of this paper • In Questions 11 – 14, show relevant mathematical reasoning and/or calculations • Marks may be deducted for careless or badly arranged work
Total marks: 70	**Section I – 10 marks** (pages 2 – 4) • Attempt Questions 1 – 10 • Allow about 15 minutes for this section **Section II – 60 marks** (pages 5 – 8) • Attempt Questions 11 – 14 • Allow about 1 hour 45 minutes for this section

Section I

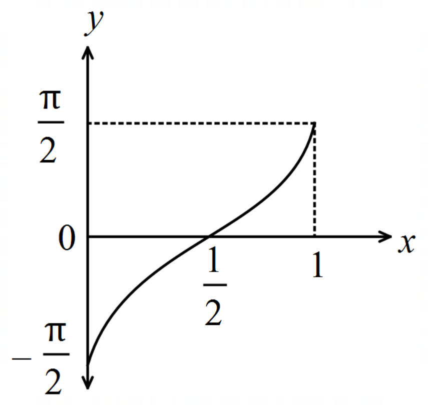
10 marks

Allow about 15 minutes for this section.

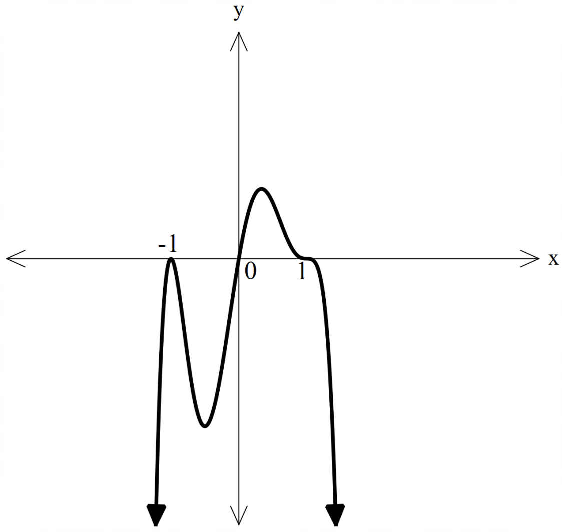
Attempt all questions.

Answer each question on the multiple choice page of your answer booklet.

1.	The polynomial $P(x) = x^4 - kx^2 - 2x + 33$ has $(x - 3)$ as a factor. What is the value of k? (A) $-\frac{40}{3}$ (B) -12 (C) 12 (D) $\frac{40}{3}$	**1**
2.	The parametric equations $x = 3 + 4\cos\theta, y = 4\sin\theta - 4$ represent a circle, with centre *C* and radius *r*. Which of the following shows the correct centre and radius? (A) $C(-3,4)$ and $r = 4$ (B) $C(3,-4)$ and $r = 4$ (C) $C(-3,4)$ and $r = 16$ (D) $C(3,-4)$ and $r = 16$	**1**
3.	The number of worms W in a worm farm at time t days after the farm is set up is given by $W = 3000e^{0.05t}$. The rate of increase in the population after 6 hours is approximately: (A) 152 worms/hour (B) 202 worms/hour (C) 152 worms/day (D) 202 worms/day	**1**
4.	If $f(x) = \frac{3+e^{2x}}{4}$, which of the following is equal to $f^{-1}(x)$? (A) $\log_e(4x - 3)$ (B) $\frac{1}{2}\log_e 4x - 3$ (C) $\log_e \frac{4x-3}{2}$ (D) $\log_e\sqrt{4x - 3}$	**1**

5.	At a family gathering, 6 adults and 3 children are randomly seated around a circular table. What is the number of possible seating arrangements if none of the children are to sit together? (A) 720 (B) 2400 (C) 14400 (D) 86400	**1**
6.	The diagram shows the graph of a function $y = f(x)$.  Which function could the graph represent? (A) $y = -\cos^{-1}(2x - 1)$ (B) $y = \sin^{-1}(2x - 1)$ (C) $y = \sin^{-1}(x - 1)$ (D) $y = -\cos^{-1}(x - 1)$	**1**
7.	$2\cos 6\theta \sin 2\theta$ is equivalent to (A) $\cos 4\theta + \cos 8\theta$ (B) $\cos 8\theta - \cos 4\theta$ (C) $\sin 8\theta + \sin 4\theta$ (D) $\sin 8\theta - \sin 4\theta$	**1**

8. Which of the following could be the equation of the graph given below? **1**



(A) $y = -\frac{x}{2}(1-x)^3(1+x)^2$

(B) $y = -\frac{x}{2}(1-x)^2(1+x)^3$

(C) $y = 2x(1+x)^3(x-1)^2$

(D) $y = 2x(1-x)^3(x+1)^2$

9. Which inequality has the same solution $|x+3| + |x-4| = 7$? **1**

(A) $|2x-1| \geq 7$

(B) $x^2 - x - 12 \leq 0$

(C) $\frac{7}{3-x} \geq 1$

(D) $\frac{1}{x-4} - \frac{1}{x+3} \leq 0$

10. What is the coefficient of x^3 in the expansion of $(1 + x + x^2)^5$? **1**

(A) 10

(B) 20

(C) 30

(D) 40

End of Section I

Section II

60 marks
Attempt questions 11-14
Allow about 1 hour 45 minutes for this section.

Answer each question on the appropriate page in the answer booklet. Extra writing paper is available.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks)

Use the Question 11 section of the writing booklet.

a) The parametric equations of a curve are $x = 4 - t,\ y = (1 - t)^2$. **2**

Find the Cartesian equation of the curve, simplifying your answer.

b) Solve the inequalities

(i) $\dfrac{2}{x-5} \geq 1$ **3**

(ii) $\dfrac{7}{9-x^2} > -1$ **3**

c) Simplify $\dfrac{\cos 2\alpha - \cos\alpha + 1}{\sin 2\alpha - \sin\alpha}$ **3**

d) A group of 12 students is to be divided into discussion groups.

(i) In how many ways can the discussion groups be formed if there are 7 people in one group and 5 people in the other? **1**

(ii) In how many ways can the discussion groups be formed if there are 3 groups containing 4 people each? **2**

(iii) In how many ways can a president and vice president be chosen for the group of 12? **1**

End of Question 11

Question 12 (15 marks)

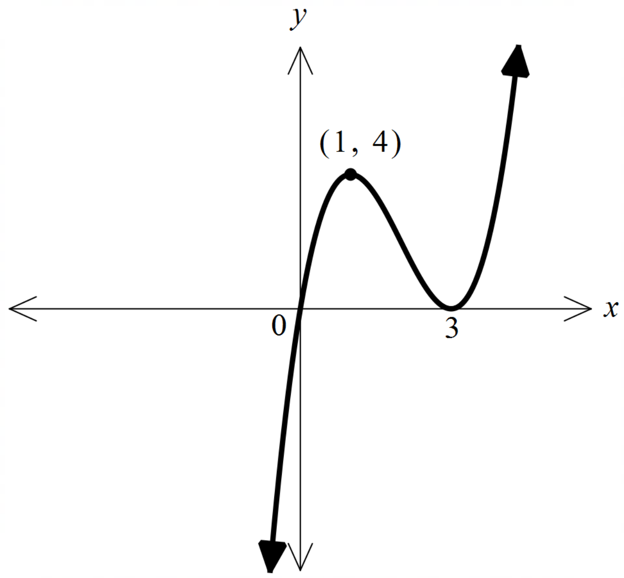
Use the Question 12 section of the writing booklet.

a) One of the roots of the equation $x^3 + kx^2 + 1 = 0$ is the sum of the other two roots.

(i) Show that $x = -\frac{k}{2}$ is a root of the equation. **2**

(ii) Find the value of k. **2**

b) The diagram shows the graph of a function $y = f(x)$



(i) Draw the graph of $y = \frac{1}{f(x)}$ showing all important features. **3**

(ii) Draw the graph of $y = f(|x|)$ showing all important features. **2**

c) Find the coefficient of x^3 in the the expansion of $(3 - 4x)^7$ **2**

d) All the letters of the word DEFINITION are arranged in a row.

(i) How many distinct arrangements are possible? **1**

(ii) In how many of these arrangements are all the vowels together? **2**

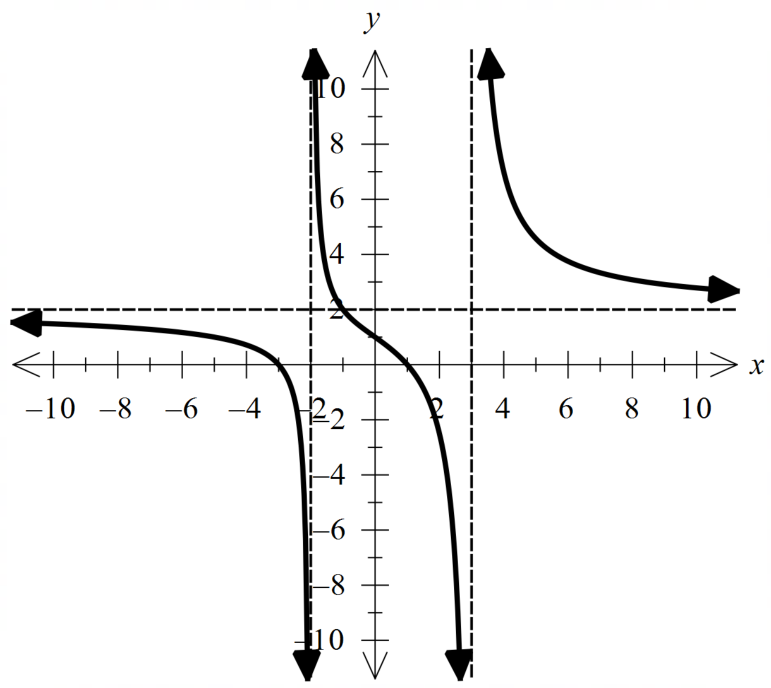
(iii) What is the probability that the letter D appears somewhere to the left of the letter T? **1**

End of Question 12

Question 13 (15 Marks)

Use the Question 13 section of the writing booklet.

a) The graph of $y = \dfrac{k(x-a)(x-b)}{(x-c)(x-d)}$ is shown, where k, a, b, c and d are constants.



(i) State the values of a, b, c and d. **2**

(ii) Find the value of k. Justify your answer. **1**

b) The equation $4x^3 - 27x + k = 0$ has a double root. Find the possible values of k. **3**

c) If $(1 + kx)^n = 1 - 12x + 60x^2 - \ldots$, find the values of k and n. **3**

d) N is the number of animals in a certain population at time t years. The population satisfies the equation $\frac{dN}{dt} = -k(N - 1000)$, for some constant k.

(i) Verify that $N = 1000 + Ae^{-kt}$ (where A is constant) is a solution of the above equation. **1**

(ii) From an initial population of 2500 animals, after 2 years the population has fallen to 2200. Determine the values for A and k. **2**

(iii) Find when the population falls to 1200. Answer in years correct to 1 decimal place. **2**

(iv) Sketch the graph of population against time. **1**

End of Question 13

Question 14 (15 Marks)

Use the Question 14 section of the writing booklet.

a) Express $\cot\frac{A}{2} - 2\cot A$ as a single trigonometric ratio. **2**

b) If α, β and γ are the roots of $3x^3 + 4x^2 - 5x - 8 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **2**

c) As a sphere of radius r is being heated, it expands such that its surface area S is increasing at a constant rate of $1.5\ \text{mm}^2\text{s}^{-1}$.

(i) Show that $\frac{dr}{dt} = \frac{3}{16\pi r}$. **2**

(ii) Hence find the rate at which its volume is increasing when its radius is 60mm. **2**

d) Find the exact value of $\cos\left[\sin^{-1}\frac{5}{13} + \tan^{-1}\frac{3}{5}\right]$. **3**

e) Consider the expression: **4**

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots. + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$$

Multiplying by x, another identity is obtained:

$$x(1+x)^n = \binom{n}{0}x + \binom{n}{1}x^2 + \binom{n}{2}x^3 + \binom{n}{3}x^4 + \ldots. + \binom{n}{n-1}x^n + \binom{n}{n}x^{n+1}$$

Find the function $F(n)$ such that

$$2\binom{n}{1} + 3\binom{n}{2} + 4\binom{n}{3} + \ldots. + n\binom{n}{n-1} = (n+2)F(n)$$

End of Examination

Year 11 Yearly 2020 SOLUTIONS.

SECTION I

1. $P(3) = 81 - 9k - 6 + 33 = 0$

$108 - 9k = 0$

$k = 12$ (C)

2. $\cos\theta = \frac{x-3}{4}$, $\sin\theta = \frac{y+4}{4}$

Since $\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x-3}{4}\right)^2 + \left(\frac{y+4}{4}\right)^2 = 1$

$(x-3)^2 + (y+4)^2 = 16$

$\therefore C(3, -4)$, $r = 4$ (B)

3. 6 hours $\Rightarrow t = 0.25$ days

Rate $= \frac{dW}{dt} = 3000e^{0.05t} \times 0.05$

$= 150e^{0.05t}$

$= 150e^{0.05 \times 0.25}$ when $t = 0.25$

$\doteq 152$ worms/day

(since t was measured in days)

(C)

4. $f: y = \frac{3+e^{2x}}{4}$

$f^{-1}: x = \frac{3+e^{2y}}{4}$

$4x - 3 = e^{2y}$

$2y = \ln(4x-3)$

$y = \frac{1}{2}\ln(4x-3)$

$= \ln\sqrt{4x-3}$ (D)

5. Arrange adults in circle $5!$

Place 3 children into the 6 spaces between adults $^6P_3$ or $6 \times 5 \times 4$

$\therefore$ No. of ways $= 5! \times {}^6P_3 = 14400$ C

6. B (can be confirmed by substituting to find endpoints)

7. From ref. sheet $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$\therefore 2\cos 6\theta \sin 2\theta = \sin 8\theta - \sin 4\theta$

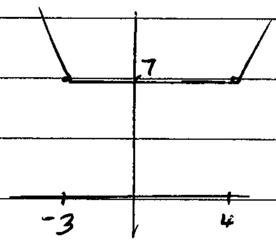
D

8. Must have factors $(x+1)^2$, x and $(x-1)^3$ and a negative leading coefficient

$\therefore y = kx(x+1)^2(x-1)^3$ where $k<0$

or $y = kx(x+1)^2(1-x)^3$ where $k>0$ D

9. $y = |x+3| + |x-4|$ Sketch



$|x+3| + |x-4| = 7$

$\leftarrow$ when $-3 \le x \le 4$

Eliminate A, since its soln will be: $\longleftarrow\!\bullet \quad \bullet\!\longrightarrow$

B is the correct soln

B

C can be eliminated since $x \neq 3$ in its solution

D can be eliminated because $x \neq 4, -3$ in its solution

10. $\left((1+x)+x^2\right)^5$

$= \binom{5}{0}(1+x)^5 + \binom{5}{1}(1+x)^4(x^2)^1 + \binom{5}{2}(1+x)^3(x^2)^2$

$+ \binom{5}{3}(1+x)^2(x^2)^3 + \binom{5}{4}(1+x)(x^2)^4 + \binom{5}{5}(x^2)^5$

$= \binom{5}{0}(1+x)^5 + \binom{5}{1}(1+x)^4x^2 + \binom{5}{2}(1+x)^3x^4 + \ldots\ldots$

The only terms above that will include x^3 in their expansions are

$\binom{5}{4}(1+x)^4x^2$ and $\binom{5}{0}(1+x)^5$

$= 5x^2(1+x)^4$ $\qquad = (1+x)^5$

Terms containing x^3 will be

$5x^2 \cdot \binom{4}{1}x + \binom{5}{3}x^3$

$= 20x^3 + 10x^3$

$= 30x^3$ $\qquad \therefore$ Coefficient $= 30$ C

1	C	6	B
2	B	7	D
3	C	8	D
4	D	9	B
5	C	10	C

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Question 11

a) $x = 4 - t \ \therefore \ t = 4 - x$

$y = (1-t)^2 \ \therefore \ y = (1 - (4-x))^2$

$= (-3 + x)^2$

$= (x-3)^2$

$= x^2 - 6x + 9$

*

(2) correct soln

(1) Substitute $t = 4 - x$

b) (i) $\dfrac{2}{x-5} \geq 1$ note: $x \neq 5$

$\times (x-5)^2$:

$2(x-5) \geq (x-5)^2$

$2x - 10 \geq x^2 - 10x + 25$

$x^2 - 12x + 35 \leq 0$

$(x-5)(x-7) \leq 0$

Roots : $x = 5, 7$

Soln : $\underline{5 < x \leq 7.}$

(3) correct soln

(2) • Solve quad. equation and attempt to find inequality

• $5 \leq x \leq 7$

(1) obtain quadratic

(ii) $\dfrac{7}{9-x^2} > -1$ note: $x \neq 3, -3$

$\times (9-x^2)^2$:

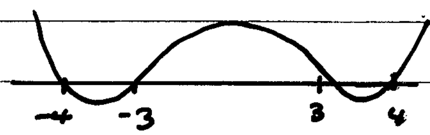
$7(9-x^2) > -(9-x^2)^2$

$(9-x^2)^2 + 7(9-x^2) > 0$

$(9-x^2)(9-x^2+7) > 0$

$(3-x)(3+x)(4-x)(4+x) > 0$

Roots $x = 3, -3, 4, -4$



Soln : $x < -4, \ -3 < x < 3, \ x > 4.$

(3) correct soln

(2) · correct roots

· solving a quartic eqn

(1) obtain a quartic eqn and attempt to factorise!

c) $\dfrac{\cos 2\alpha - \cos\alpha + 1}{\sin 2\alpha - \sin\alpha} = \dfrac{2\cos^2\alpha - 1 - \cos\alpha + 1}{2\sin\alpha\cos\alpha - \sin\alpha}$

$= \dfrac{\cos\alpha\,(2\cos\alpha - 1)}{\sin\alpha\,(2\cos\alpha - 1)}$

$= \dfrac{\cos\alpha}{\sin\alpha}$

$= \cot\alpha$.

(3) correct solution

(2) • correct formulae for $\cos 2\alpha$ and $\sin 2\alpha$

• a correct formula for $\cos 2\alpha$ or $\sin 2\alpha$ and attempt to simplify

(1) correct formula for $\cos 2\alpha$ or $\sin 2\alpha$

d) (i) ${}^{12}C_7 \times {}^{5}C_5$ (or just ${}^{12}C_7$) $= 792$

(1) correct answer

(ii) $\dfrac{{}^{12}C_4 \times {}^{8}C_4 \times {}^{4}C_4}{3!} = \dfrac{34\,650}{6}$

$= 5775$

(2) correct answer

(1) correct numerator

(iii) $12 \times 11 = 132$

(1) correct answer

Q12.

a) (i) Let roots $= \alpha, \beta, \alpha+\beta$.

Sum of roots $= \alpha + \beta + (\alpha+\beta)$

$= 2\alpha + 2\beta$

$= 2(\alpha+\beta) = -k$

$\therefore \alpha+\beta = -\frac{k}{2}$

Since $\alpha+\beta$ is one of the roots, $-\frac{k}{2}$ is one of the roots.

(2) correctly shown

(1) Attempts to use sum of roots or substitutes $x = -\frac{k}{2}$.

(ii) Product of roots $\alpha\beta(\alpha+\beta) = -1$

$\alpha\beta\left(-\frac{k}{2}\right) = -1$

$\alpha\beta = \frac{2}{k}$

Sum of pairs $\alpha\beta + \alpha(\alpha+\beta) + \beta(\alpha+\beta) = 0$

$\frac{2}{k} + (\alpha+\beta)^2 = 0$

$\frac{2}{k} + \left(-\frac{k}{2}\right)^2 = 0$

$\frac{k^2}{4} = -\frac{2}{k}$

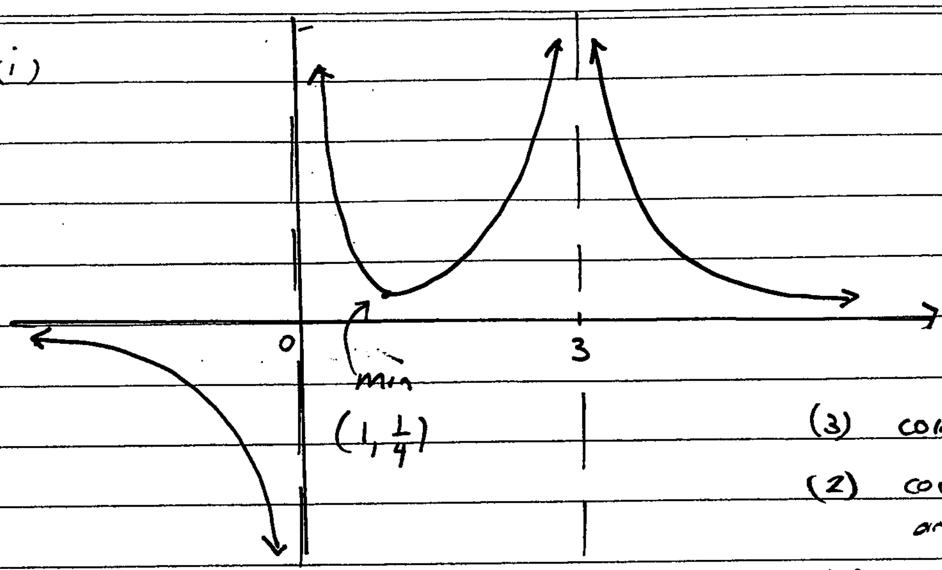
$k^3 = -8$

$\underline{k = -2}$

(2) correct sol^n

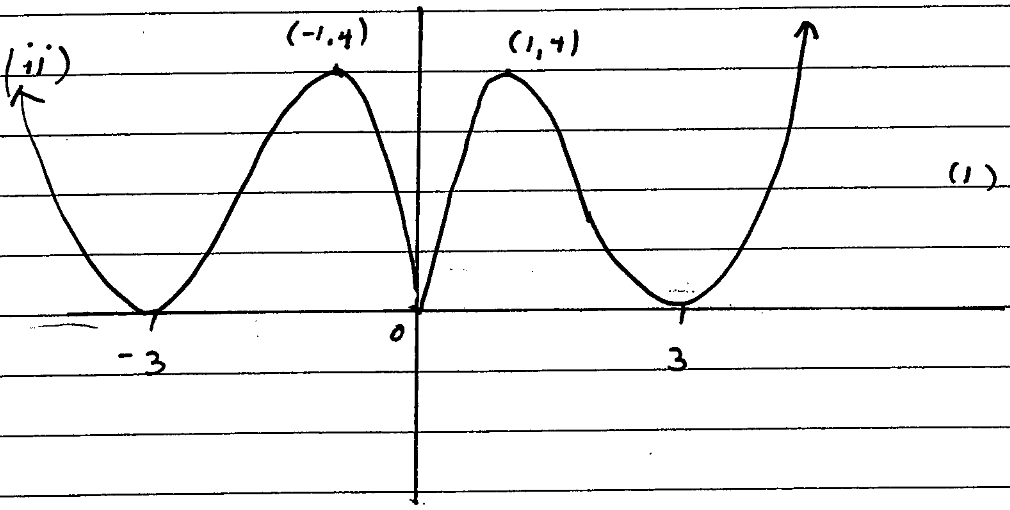
(1) attempts to use sums + product.

b) (i)



(3) correct graph (include turning point),
(2) correct asymptotes and shape
(1) correct asymptotes

(ii)



(1) correct graph, include (1,4) and (-1,4)

c) $\binom{7}{3} \cdot 3^4 \cdot (-4x)^3 = 35 \times 81 \times -64x^3$

$\therefore$ Coefficient $= -181\,440$

(2) correct soln
(1) partially correct method.

d) DEFINITION

10 letters

3I, 2N

5 vowels

5 consonants.

(i) $\dfrac{10!}{3!\,2!} = \underline{302400}$ (1) correct answer

(ii) Arrange consonants (D, F, N, T, N) $\dfrac{5!}{2!} = 60$

^ _ ^ _ ^ _ ^ _ ^ _ ^

Place the vowels group in one of the 6 places indicated 6

Arrange the vowels within their group $\dfrac{5!}{3!}$

(1) correct answer

(1) sig. progress

$\therefore\ 60 \times 6 \times \dfrac{5!}{3!} = 60 \times 6 \times 20$

$= \underline{7200}$

(iii) The letter D can either appear to the left or to the right of the letter N, with equal probabilities.

$\therefore$ Probability $= \underline{\tfrac{1}{2}}$. (1) correct answer.

Question 13.

a) (i) $a = -3$, $b = 1$ (or vice versa) (2) All 4 correct

$c = -2$, $d = 3$ (or vice versa) (1) One pair correct

· Signs opp. but otherwise correct

(ii) $y = \dfrac{k(x-3)(x+1)}{(x-2)(x+3)}$

Sub. $(0,1)$:

$$1 = \frac{k \times -3 \times 1}{-2 \times 3}$$

$$-6 = -3k$$ (1) correct soln + justification.

$$\underline{k = 2}$$

b) Let $P(x) = 4x^3 - 27x + k = 0$

If $P(x) = 0$ has a double root
then $P'(x) = 12x^2 - 27 = 0$ has a single root $\Big\}$ in common

$P'(x) = 3(4x^2 - 9)$

$= 3(2x-3)(2x+3)$

$= 0$ when $x = \frac{3}{2}, -\frac{3}{2}$

If $x = \frac{3}{2}$, then $4\left(\frac{27}{8}\right) - 27\left(\frac{3}{2}\right) + k = 0$

$\therefore \underline{k = 27}$

If $x = -\frac{3}{2}$, then $4\left(-\frac{27}{8}\right) - 27\left(-\frac{3}{2}\right) + k = 0$

$\underline{k = -27}$

$\therefore$ The TWO possible values are $\underline{k = 27, -27}$

(3) correct soln

(2) find $x = \pm\frac{3}{2}$ but not both solns given

(1) recognise common root exists and find $P'(x)$.

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c) From T_2 : $\binom{n}{1}(kx)^1 = -12x$

$$nkx = -12x$$

$$nk = -12 \qquad ①$$

From T_3 : $\binom{n}{2}(kx)^2 = 60x^2$

$$\binom{n}{2}k^2 = 60$$

$$\frac{n(n-1)}{2} \cdot k^2 = 60$$

$$k^2 n(n-1) = 120 \qquad ②$$

From ①, $k = \frac{-12}{n}$

Into ② $\left(\frac{-12}{n}\right)^2 \cdot n(n-1) = 120$

$$\frac{144}{n^2} \cdot n(n-1) = 120$$

$$144n^2 - 144n = 120n^2$$

$$24n^2 - 144n = 0$$

$$24n(n-6) = 0$$

$$\cancel{n=0} \quad \text{or} \quad n = 6 \qquad (n>0)$$

$\therefore$ Since $nk = -12$, $6k = -12$

$$\underline{k = -2}$$

(3) correct soln
(2) sig. progress in solving simult eqns
(1) obtain ① and ② or equivalent.

~~If $N = 1000 + Ae^{-kt}$, then:~~

d) (i) By differentiation, $\frac{dN}{dt} = \frac{d}{dt}\left(1000 + Ae^{-kt}\right)$

$$= 0 + \left(Ae^{-kt}\right) \times -k$$

and $Ae^{-kt} = N - 1000$

$$= -k(N-1000)$$

(1) correctly shown

(ii) $N = 1000 + Ae^{-kt}$

$t=0$	$2500 = 1000 + Ae^0$
$N = 2500$	$1500 = A$

$\therefore N = 1000 + 1500e^{-kt}$

$t = 2$ $N = 2200$	$2200 = 1000 + 1500e^{-2k}$

$$0.8 = e^{-2k}$$

$$-2k = \ln 0.8$$

$$k = \underbrace{-\tfrac{1}{2}\ln 0.8}_{\text{Store this value for accuracy.}} \doteqdot 0.1116$$

(2) correct A and k
(1) correct A or k

(iii) $N = 1000 + 1500e^{-0.1116\ldots t}$

If $N = 1200$,

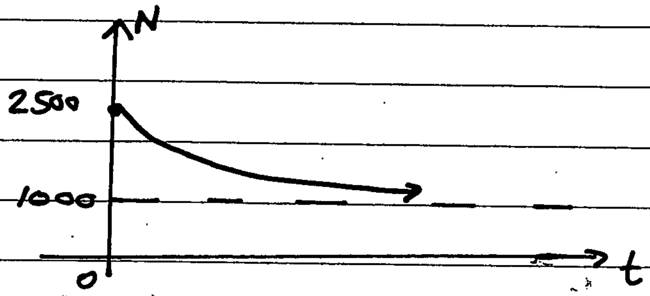
$$1200 = 1000 + 1500e^{-0.1116\ldots t}$$

$$\frac{2}{15} = e^{-0.1116\ldots t}$$

(2) correct soln
(1) obtain $e^{-kt} = \frac{2}{15}$

$$t = -\frac{1}{k}\ln\frac{2}{15} \doteqdot 18.1 \text{ years.}$$

(iv)



(1) correct graph with initial value and asymptote

Question 14.

a) $\cot\frac{A}{2} - 2\cot A$

$= \frac{1}{t} - \cancel{2}\left(\frac{1-t^2}{\cancel{2}t}\right)$ where $t = \tan\frac{A}{2}$

$= \frac{1-(1-t^2)}{t}$

$= \frac{t^2}{t}$

$= t$

$= \tan\frac{A}{2}$.

(2) correct soln
(1) attempt to use $\tan A = \frac{2t}{1-t^2}$

b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$

$= \frac{-5/3}{8/3}$

$= -5/8$.

(2) correct soln
(1) correct alg. fraction + correct product or sum of pairs.

c) (i) $\frac{dS}{dt} = \frac{dS}{dr}\cdot\frac{dr}{dt}$ $\left(S = 4\pi r^2, \frac{dS}{dr} = 8\pi r\right)$

$1.5 = 8\pi r \times \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1.5}{8\pi r} = \frac{3}{16\pi r}$.

(2) correctly shown
(1) forms a correct eqn with derivatives

(ii) $\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt}$ $\left(V = \frac{4}{3}\pi r^3, \frac{dV}{dr} = 4\pi r^2\right)$

$\frac{dV}{dt} = 4\cancel{\pi}\cancel{r^2} \times \frac{3}{16\cancel{\pi}\cancel{r}}$

$= \frac{3r}{4}$

$= \frac{3\times 60}{4}$ when $r = 60$ mm

$= 45$ mm^3/s.

(2) correct soln
(1) correct eqn for $\frac{dV}{dt}$ in terms of r

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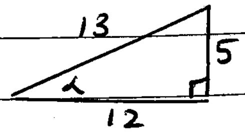
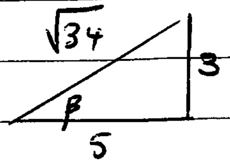
d) $\cos\left(\underbrace{\sin^{-1}\frac{5}{13}}_{\alpha} + \underbrace{\tan^{-1}\frac{3}{5}}_{\beta}\right)$





$= \cos(\alpha+\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{12}{13}\cdot\frac{5}{\sqrt{34}} - \frac{5}{13}\cdot\frac{3}{\sqrt{34}}$

$= \frac{45}{13\sqrt{34}}$

(3) correct soln

(2) attempts to apply $\cos(\alpha \pm \beta)$ with values from Δs

(1) Δs with sides as shown.

e) Differentiating both sides of the 2nd identity:

(4) correct soln

(3) correctly substitutes $x=1$ into 2 correct expressions

(2) • Correctly differentiates both sides
• Sub $x=1$, but only one side correct

(1) Correctly differentiates one side

$LHS = x.n(1+x)^{n-1} + (1+x)^n . 1$

$= (1+x)^{n-1}(nx + 1 + x)$

$RHS = 1 + 2\binom{n}{1}x + 3\binom{n}{2}x^2 + 4\binom{n}{3}x^3 + \dots + n\binom{n}{n-1}x^{n-1}$

$+ (n+1)\binom{n}{n}x^n$

Substitute $x=1$ into both sides :

$(1+1)^{n-1}(n+1+1) = \underline{\underline{1}} + 2\binom{n}{1} + 3\binom{n}{2} + 4\binom{n}{3} + \dots + n\binom{n}{n-1}$

$+ \underline{(n+1)}.1$

$2^{n-1}(n+2) = 2\binom{n}{1} + 3\binom{n}{2} + 4\binom{n}{3} + \dots + n\binom{n}{n-1}$

$+ n + 2.$

$\therefore 2\binom{n}{1} + 3\binom{n}{2} + 4\binom{n}{3} + \dots + n\binom{n}{n-1}$

$= 2^{n-1}(n+2) - (n+2)$

$= (n+2)(2^{n-1} - 1) \quad \therefore \underline{F(x) = 2^{n-1} - 1}$