
Student Number

Barker College

2024 YEAR 11
PRELIMINARY EXAMINATION

Mathematics Advanced

Staff Involved:

- DZB • SJB • JZT • DZP • ALY • ARP
- ECB* • FHD • RJW • DXC • AYH • BHC

Tuesday 10 September 2024

210 copies

TOPICS COVERED:		
Algebra, Number and Surds	Functions and Graphs	Transformations and Symmetry
The Coordinate Plane	Trigonometry	Exponential and Logarithmic Functions
Differentiation and Curve Sketching	Extending Calculus	Probability

General Instructions:

- Reading time - 5 minutes
- Working time - 2 hours 30 minutes
- Write your Student Number on all answer pages
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations

Total marks: 100

Section I - 10 marks (pages 2 - 5)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 9 - 28)

- Attempt Questions 11 - 20
- Show all necessary working
- Allow about 2 hours and 15 minutes for this section

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

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1 Determine the gradient of the line perpendicular to $y = \frac{x}{2} - 5$.

A. -2

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 2

2 What is the value of $log_{31}586$ correct to **three significant figures**?

A. 1.856

B. 1.86

C. 2.768

D. 2.77

3 Given that $log_a 4 = 0.420$ and $log_a 5 = 0.488$. What is the value of $log_a 100$?

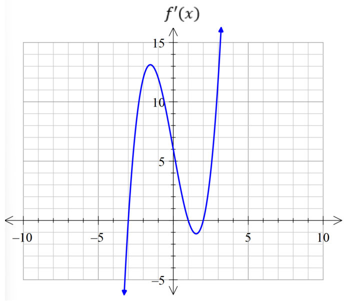
A. 0.410

B. 0.664

C. 1.396

D. 2.908

4 The graph of the gradient function, $f'(x)$, is shown below. For what value(s) of x does the function, $f(x)$, have stationary points?



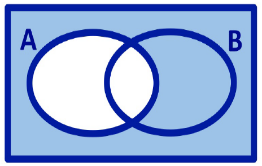
A. $x = -1.5$ and $x = 1.5$

B. $x = -3$, $x = 1$ and $x = 2$

C. $x = -3$, $x = -1.5$, $x = 1$, $x = 1.5$ and $x = 2$

D. $x = 1.5$ only

5 Determine the set notation represented by the **shaded area** in the Venn diagram below.



A. B'

B. A'

C. $A' \cap B$

D. $B \cup (A' \cap B')$

6 Determine the value(s) of k for which $|2k - 1| = k$

A. 1 only

B. $-\frac{1}{3}$ only

C. -1 or $-\frac{1}{3}$

D. 1 or $\frac{1}{3}$

7 Determine the value of p so that $\frac{a^2 a^{-3}}{\sqrt{a}} = a^p$.

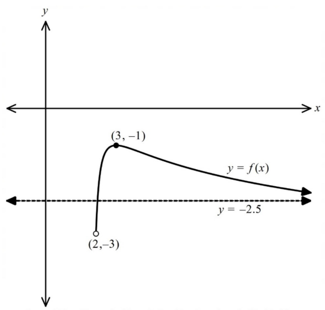
A. -3

B. 12

C. $-\frac{1}{2}$

D. $-\frac{3}{2}$

8 The graph of a function $y = f(x)$ is shown below. It has an asymptote at $y = -2.5$. Which statement **correctly** describes its domain and range?



A. Domain: $(2, \infty)$; Range $[-2.5, -1]$

B. Domain: $(-3, -1]$; Range $(2, \infty)$

C. Domain: $[2, \infty)$; Range $[-3, -1)$

D. Domain: $(2, \infty)$; Range $(-3, -1]$

9 What is the **area**, in cm^2, of the sector with radius 8 cm and arc length 20π cm?

A. 80π

B. 80

C. 160π

D. 20π

10 Given that f and g are two functions such that $f(x) = 2x$ and $g(x + 2) = 3x + 1$, which function is $f(g(x))$?

A. $6x - 5$

B. $6x + 1$

C. $6x - 10$

D. $6x + 2$

End of Section I

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Student Number

Barker College

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11 - 20
Allow about 2 hours and 15 minutes for this section

Instructions:

- Write your Student Number at the top of the page where indicated.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing booklets are available.

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Student Number

Question 11 (9 marks)

(a) **Solve** this pair of simultaneous equations: **3**

$$3x + y = 6$$
$$2x - 3y = 15$$

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(b) **Solve** $3 - 5x \leq 2$ **1**

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(c) If $f(x) = x^2 - 2x + 5$, find $\frac{1}{2}(f(-2) - f(2))$ **2**

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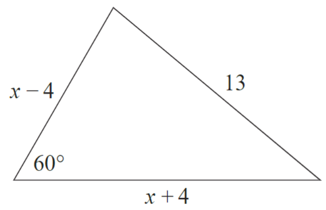
(d) Calculate the value of x in the diagram below. **3**



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Student Number

Question 12 (9 marks)

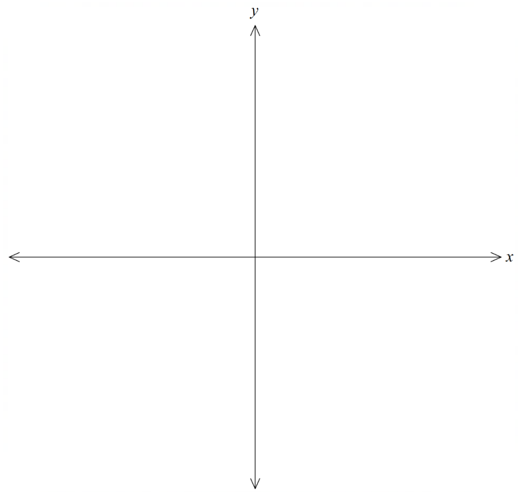
(a) **Simplify** $3^{2m} \times 3^{m-2} \times 3^{m}$ **1**

(b) State the **domain** of the function $f(x) = \sqrt{x}$ **1**

(c) Determine the equation of the line that is **parallel** to the interval joining the points $A(3, 5)$ and $B(6, 10)$, and passes through the point $C(-3, -1)$. **3**
Leave your answer in **general form**.

(d) **Prove** whether $f(x) = \frac{x^4-1}{x}$ is even, odd or neither. **2**

(e) Sketch the graph of $y - 2 = -2e^x$, **showing all intercept(s) and asymptote(s).** **2**



Student Number

Question 13 (9 marks)

(a) By first expressing in **index form**, solve $log_5(2x+1) = 2$, for x. **2**

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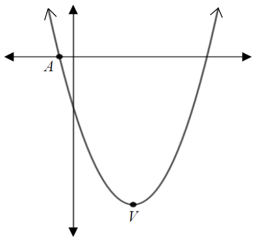
(b) The diagram shows the graph of $y = x^2 - 4x - 2$.



i) Find the coordinate of the vertex, V. **2**

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ii) Find the x-coordinate of the intercept marked A, in **exact form**. **2**

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(c) The number of hours H that is take for a block of ice to melt varies inversely with the temperature T. At 32°C it takes 6 hours for a block of ice to melt.

i) Find an **equation** for the graph in terms of H and T in the form $H = \frac{k}{T}$, where k is a constant. **1**

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ii) How much longer will it take the same size block of ice to melt at 10°C, compared to 25°C? Leave your answer to the **nearest minute.** **2**

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Student Number

Question 14 (9 marks)

(a) Use the method of **first principles** to differentiate $f(x) = x^2 + 4x$, using: **3**

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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(b) Find the quadratic function $g(x)$ such that $g(0) = 16$ and $g(4) = g(-2) = 0$. **3**

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(c) A packet of lollies contains 5 red lollies and 14 green lollies. Two lollies are selected at random **without replacement**.

i) Draw a **probability tree diagram** to illustrate the possible outcomes. **2**

ii) What is the probability that the two lollies are **different colours**? **1**

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Student Number

Question 15 (9 marks)

(a) Differentiate and **simplify** the following functions with respect to x:

i) $f(x) = 2x^5 + 3x^2 - x + 1$ **1**

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ii) $f(x) = x^3 e^{2x}$ **2**

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iii) $f(x) = \frac{5x^2}{x^2+1}$ **2**

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iv) $f(x) = 3\sqrt{x} - \frac{4}{x^7}$ **2**

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(b) Sketch the graph of $f(x) = -(x-1)^2(x+2)(x+5)$, showing **ONLY** the x **and** y intercepts. **2**

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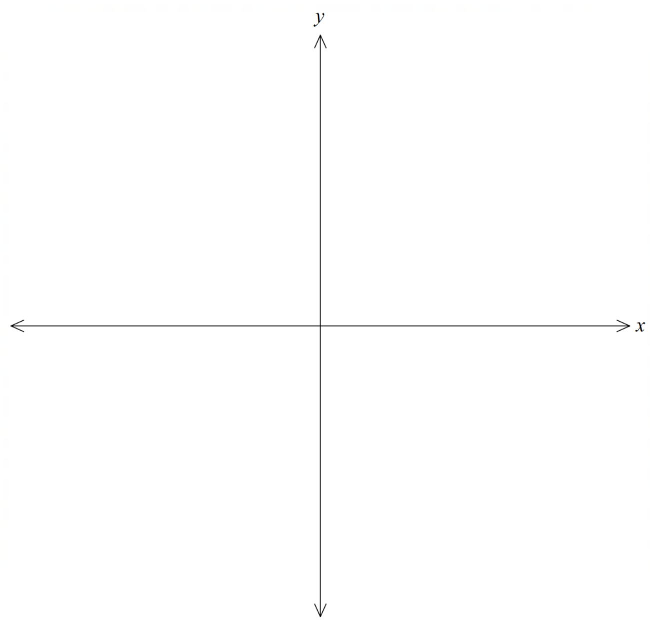

Student Number

Question 16 (9 marks)

(a) Determine the solution(s) for the following equations in the interval $0 \leq \theta \leq 2\pi$

i) $2\sin\theta = -\sqrt{3}$ **3**

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ii) $2\cos^2\theta - \cos\theta = 0$ **4**

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(b) **Prove** that $\sec x - \cos x = \sin x \tan x$ **2**

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Student Number

Question 17 (9 marks)

(a) A circle has equation $x^2 - 6x + y^2 + 4y + 9 = 0$

i) Determine the **centre** and **radius** of the circle. **2**

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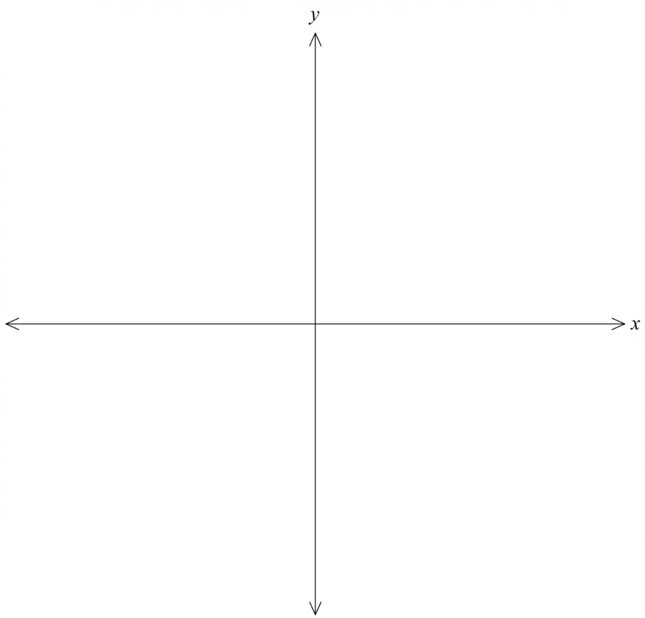
ii) The circle is reflected across the x axis. **Sketch the reflected circle**, showing the coordinates of the **centre** and the **radius.** **1**



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(b) Find the **equation** of the tangent to the curve $y = (2x+1)^4$ at the point where $x = -1$. **3**

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(c) A survey of the reading habits of some students revealed that, 25% read quality newspapers, 45% read tabloid newspapers and 40% do not read newspapers at all.

A student is selected at random. Given that this student reads newspapers, find the probability that this student **only** reads quality newspapers. **3**

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Student Number

Question 18 (9 marks)

(a) The population of native penguins on an island has been studied since 1950 for conservation purposes.
The number of penguins, P, can be modelled by the equation $P = 2500e^{-0.036t}$, where t is the number of years after 1950.

i) What was the population of native penguins in 1960? **1**

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ii) Show that the penguin population was declining at a **rate** of approximately 44 penguins per year in 1970. **2**

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iii) When the penguin population reaches 250, they are listed as being in danger of extinction. After **how many years** will this occur? **2**

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(b) **Each week** Percy and Kura take part in a raffle at their respective workplaces. The probability that Percy wins a prize in his raffle is $\frac{1}{9}$. The probability that Kura wins a prize in his raffle is $\frac{1}{16}$.
What is the probability that, during the next three weeks, **at least** one of them wins a prize? **2**

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(c) Given that $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-3}$, determine the equation of $y = f(g(x))$. **2**

State the **domain** of $f\big(g(x)\big)$.

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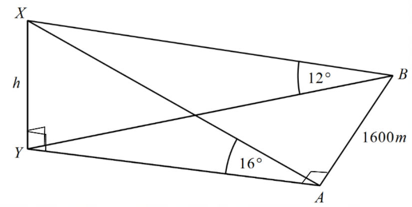
Question 19 (9 marks)

(a) Determine the value(s) of m for which the equation $3x^2 - 2mx = -3$ has **one real solution**. **2**

(b) **Solve** for x in the following: **3**

$$log_4(4x + 16) - log_4(x^2 - 2) = 1$$

(c) Hunter walks 1600 m due North along a road from point A to point B.
The point A is due east of a hill XY, where X is the top of the hill.
The point Y is directly below point X and is on the same horizontal plane as the road.
Let the height of the hill above point Y be h metres.
From point A, the angle of elevation to the top of the hill is $16°$.
From point B, the angle of elevation to the top of the hill is $12°$.



i) **Show** that $BY = h \cot 12°$ **1**

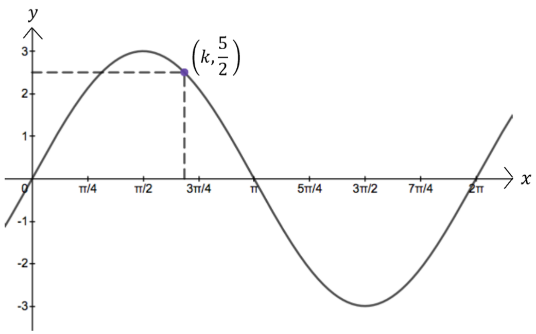
ii) Hence, find the value of h, **correct to 2 decimal places**. **3**

Student Number

Question 20 (9 marks)

(a) **Solve** $\sin\theta - \sqrt{3}\cos\theta = 0$ for $0° \le \theta \le 360°$ **2**

(b) The graph represents the function $y = 3\sin x$



The point $\left(k, \frac{5}{2}\right)$ lies on the graph as shown. **3**
Find the **exact value** of $\tan k$.

(c) The graph of the curve $y = a + bx + cx^2 + dx^3$ has a turning point at $(0, 0)$, where a, b, c and d are real numbers. **4**
As the curve passes through the point $\left(\frac{5}{2}, 0\right)$, it slopes downwards at an angle of $45°$ with the x-axis.

Determine the **equation** of the curve.

End of Paper

Year 11 | Y11 Prelim Exam Student Solutions | Page 1

1
$y = \frac{x}{2} - 5$
$= \frac{1}{2}x - 5$
$m = \frac{1}{2}$
$\perp m = -2$ (A)

2
$\log_{31}(586) = 1.86$ (B)

3
$\log_a(100) = \log_a(5^2 \times 4)$
$= \log_a(5^2) + \log_a(4)$
$= 2\log_a(5) + \log_a(4)$
$= 2(0.488) + (0.420)$
$= 1.396$ (C)

4
Stat pts. occur when $f'(x) = 0$
$\therefore$ at $x = -3$, $x = 1$ and $x = 2$ (B)

5
A' (B)

6
$|2k - 1| = k$
$2k - 1 = k$ or $2k - 1 = -k$
$k = 1$ $\quad$ $3k = 1$
$k = \frac{1}{3}$
$\therefore k = 1$ or $k = \frac{1}{3}$ (D)

7
$\frac{a^2 a^{-3}}{\sqrt{a}} = \frac{a^{2+(-3)}}{a^{\frac{1}{2}}}$
$= \frac{a^{-1}}{a^{\frac{1}{2}}}$
$= a^{-1-\frac{1}{2}}$
$= a^{-\frac{3}{2}}$
$\therefore p = -\frac{3}{2}$ (D)

8
D: $(2, \infty)$
R: $(-3, -1]$
(D)

9
$l = r\theta$
$20\pi = 8\theta$
$\theta = \frac{20\pi}{8} = \frac{5\pi}{2}$
$A = \frac{1}{2}r^2\theta$
$= \frac{1}{2}(8)^2\left(\frac{5\pi}{2}\right)$
$= 80\pi \text{ cm}^2$ (A)

10
$g(x+2) = 3x + 1$
$g((x+2) - 2) = 3(x-2) + 1$
$g(x) = 3x - 6 + 1$
$= 3x - 5$
$f(g(x)) = 2(3x - 5)$
$= 6x - 10$ (C)

11
(a)
$3x + y = 6$
$\times 3$
$2x - 3y = 15$ ①
$9x + 3y = 18$ ②
① + ②
$11x = 33$
$x = 3$
Sub $x = 3$ into ②
$2(3) - 3y = 15$
$-3y = 9$
$y = -3$
$\therefore x = 3$ and $y = -3$

Year | | Page 2

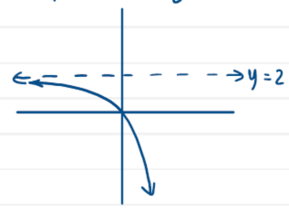
(b) $3 - 5x \leq 2$
$-5x \leq -1$
$x \geq \frac{1}{5}$

(c) $f(-2) = (-2)^2 - 2(-2) + 5$
$= 13$
$f(2) = (2)^2 - 2(2) + 5$
$= 5$
$\frac{1}{2}(f(-2) - f(2)) = \frac{1}{2}(13 - 5)$
$= 4$

(d) $13^2 = (x-4)^2 + (x+4)^2 - 2(x-4)(x+4)\cos(60)$
$169 = x^2 - 8x + 16 + x^2 + 8x + 16 - 2(x^2 - 16)\left(\frac{1}{2}\right)$
$= 2x^2 + 32 - x^2 + 16$
$= x^2 + 48$
$x^2 = 121$
$x = \pm 11$
However $x - 4 > 0 \Rightarrow x > 4$
$\therefore x = 11$

(d) $f(x) = \frac{x^4 - 1}{x}$
$f(-x) = \frac{(-x)^4 - 1}{(-x)}$
$= \frac{x^4 - 1}{-x}$
$-f(x) = -\left(\frac{x^4 - 1}{x}\right)$
$= \frac{x^4 - 1}{-x}$
$f(-x) = -f(x)$
$\therefore$ ODD

(e) $y - 2 = -2e^x$
$y = -2e^x + 2$
At $x = 0$
$y = -2e^{(0)} + 2$
$= 0$
$\therefore$ passes through $(0, 0)$



12
(a) $3^{2m+m-2+m}$
$= 3^{4m-2}$

(b) $x \geq 0$

(c) $m_{AB} = \frac{10-5}{6-3}$
$= \frac{5}{3}$
Passes through $C(-3, -1)$, $m = \frac{5}{3}$
$y - (-1) = \frac{5}{3}(x - (-3))$
$y + 1 = \frac{5}{3}(x + 3)$
$3y + 3 = 5x + 15$
$5x - 3y + 12 = 0$

13
(a) $\log_5(2x + 1) = 2$
$2x + 1 = 5^2$
$2x + 1 = 25$
$2x = 24$
$x = 12$

Year | | Page 3

(b) (i) $x = -\frac{b}{2a}$
$= -\frac{(-4)}{2(1)}$
$= 2$

At $x=2$
$y = (2)^2 - 4(2) - 2$
$= -6$
$\therefore V(2, -6)$

(ii) $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)}$
$= 2 \pm \sqrt{6}$
$\therefore$ for A, $x = 2 - \sqrt{6}$

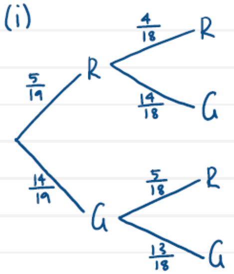
(c) (i) $H = \frac{k}{T}$
At $T = 32$, $H = 6$
$6 = \frac{k}{32}$
$k = 192$
$\therefore H = \frac{192}{T}$

(ii) At $T = 10$
$H = \frac{192}{10} = 19.2$
At $T = 25$
$H = \frac{192}{25} = 7.68$
Difference $= 19.2 - 7.68$
$= 11.52$
$= 11$ hours 31 min

14 (a) $f(x) = x^2 + 4x$
$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - (x^2+4x)}{h}$
$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 4x + 4h - x^2 - 4x}{h}$
$= \lim_{h \to 0} \frac{2xh + h^2 + 4h}{h}$
$= \lim_{h \to 0} (2x + h + 4)$
$= 2x + (0) + 4$
$= 2x + 4$

(b) $g(x) = ax^2 + bx + c$
$g(0) = 16$
$16 = a(0)^2 + b(0) + c$
$c = 16$
$g(4) = 0$
$0 = a(4)^2 + b(4) + 16$
$0 = 16a + 4b + 16$
$0 = 4a + b + 4$ (1)
$g(-2) = 0$
$0 = a(-2)^2 + b(-2) + 16$
$0 = 4a - 2b + 16$
$0 = 2a - b + 8$ (2)
(1) + (2)
$0 = 6a + 12$
$a = -2$
Sub $a = -2$ into (1)
$0 = 4(-2) + b + 4$
$b = 4$
$\therefore g(x) = -2x^2 + 4x + 16$

(c) (i)



(ii) P(different colours)
$= P(RG) + P(GR)$
$= \left(\frac{5}{19} \times \frac{4}{18}\right) + \left(\frac{14}{19} \times \frac{5}{18}\right)$
$= \frac{70}{171}$

Year | | Page 4

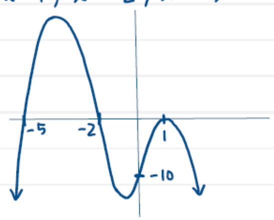
15 (a) (i) $f'(x) = 10x^4 + 6x - 1$

(ii) $u = x^3$ $\quad v = e^{2x}$
$u' = 3x^2$ $\quad v' = 2e^{2x}$
$f'(x) = uv' + vu'$
$= (x^3)(2e^{2x}) + (e^{2x})(3x^2)$
$= 2x^3e^{2x} + 3x^2e^{2x}$

(iii) $u = 5x^2$ $\quad v = x^2 + 1$
$u' = 10x$ $\quad v' = 2x$
$f'(x) = \frac{vu' - uv'}{v^2}$
$= \frac{(x^2+1)(10x) - (5x^2)(2x)}{(x^2+1)^2}$
$= \frac{10x^3 + 10x - 10x^3}{(x^2+1)^2}$
$= \frac{10x}{(x^2+1)^2}$

(iv) $f(x) = 3x^{\frac{1}{2}} - 4x^{-7}$
$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + 28x^{-8}$
$= \frac{3}{2\sqrt{x}} + \frac{28}{x^8}$

(b) y-int at $f(0)$
$f(0) = -(0-1)^2(0+2)(0+5)$
$= -10$
x-int when $f(x) = 0$
$0 = -(x-1)^2(x+2)(x+5)$
$x = 1$, $x = -2$, $x = -5$



16 (a) (i) $2\sin\theta = -\sqrt{3}$
$\sin\theta = -\frac{\sqrt{3}}{2}$
r.a.∠ $= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
$= \frac{\pi}{3}$

$\theta = \pi + \frac{\pi}{3}$, $2\pi - \frac{\pi}{3}$
$\theta = \frac{4\pi}{3}$, $\frac{5\pi}{3}$

(ii) $2\cos^2\theta - \cos\theta = 0$
$\cos\theta(2\cos\theta - 1) = 0$
$\cos\theta = 0$ or $2\cos\theta - 1 = 0$
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad \cos\theta = \frac{1}{2}$
r.a.∠ $= \cos^{-1}\left(\frac{1}{2}\right)$
$= \frac{\pi}{3}$

$\theta = \frac{\pi}{3}$, $2\pi - \frac{\pi}{3}$
$\theta = \frac{\pi}{3}$, $\frac{5\pi}{3}$
$\therefore \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{5\pi}{3}$

(b) LHS $= \sec x - \cos x$
$= \frac{1}{\cos x} - \cos x$
$= \frac{1}{\cos x} - \frac{\cos x}{1}$
$= \frac{1 - \cos^2 x}{\cos x}$
$= \frac{\sin^2 x}{\cos x}$
$= \sin x \times \frac{\sin x}{\cos x}$
$= \sin x \tan x$
$=$ RHS

17 (a)(i) $x^2-6x+y^2+4y+9=0$

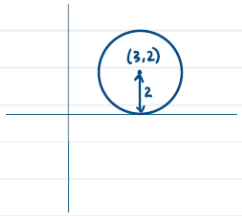
$x^2-6x+y^2+4y=-9$

$x^2-6x+9+y^2+4y+4=-9+9+4$

$(x-3)^2+(y+2)^2=4$

$\therefore$ centre $(3,-2)$ and $r=2$

(ii)



(b) $y=(2x+1)^4$

$y'=4(2x+1)^3(2)$

$=8(2x+1)^3$

At $x=-1$

$y=(2(-1)+1)^4=1$

$y'=8(2(-1)+1)^3=-8$

At $(-1,1) \Rightarrow m=-8$

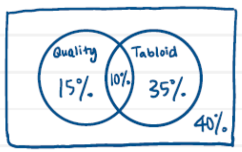
$y-1=-8(x-(-1))$

$=-8(x+1)$

$=-8x-8$

$y=-8x-7$

(c)



P(only Quality | reads newspapers)

$=\frac{15\%}{60\%}$

$=25\%$

18 (a)(i) At $t=10$

$P=2500e^{-0.036(10)}$

$=1744$

(ii) $\frac{dP}{dt}=2500\times-0.036e^{-0.036t}$

$=-90e^{-0.036t}$

At $t=20$

$\frac{dP}{dt}=-90e^{-0.036(20)}$

$=-43.81\ldots$

$\therefore$ declining 44 penguins/year

(iii) When $P=250$

$250=2500e^{-0.036t}$

$0.1=e^{-0.036t}$

$-0.036t=\ln(0.1)$

$t=\frac{\ln(0.1)}{-0.036}$

$=63.96$

$\therefore$ after 64 years

(b) P(at least one wins)

$=1-P(\text{none wins})$

$=1-\left(\frac{8}{9}\times\frac{15}{16}\right)^3$

$=\frac{91}{216}$

(c) $f(g(x))=(\sqrt{x-3})^2+1$

$=x-3+1$

$=x-2$

$x-3\geq 0$

$x\geq 3$

19 (a) $3x^2-2mx=-3$

$3x^2-2mx+3=0$

$\Delta=b^2-4ac$

$=(-2m)^2-4(3)(3)$

$=4m^2-36$

one real solution $\Rightarrow \Delta=0$

$0=4m^2-36$

$0=m^2-9$

$m^2=9$

$m=\pm 3$

(b) $\log_4(4x+16)-\log_4(x^2-2)=1$

$\log_4\left(\frac{4x+16}{x^2-2}\right)=1$

$\frac{4x+16}{x^2-2}=4^1$

$4x+16=4(x^2-2)$

$=4x^2-8$

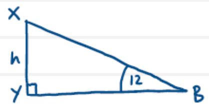
$4x^2-4x-24=0$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3$ or $x=-2$

(c) (i)



$\tan(12)=\frac{h}{BY}$

$BY=\frac{h}{\tan(12)}$

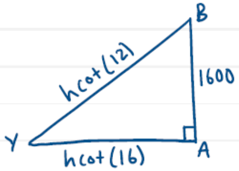
$=h\cot(12)$

(ii)

$\tan(16)=\frac{h}{AY}$

$AY=\frac{h}{\tan(16)}$

$=h\cot(16)$



$(h\cot 12)^2=(h\cot 16)^2+1600^2$

$h^2\cot^2 12-h^2\cot^2 16=1600^2$

$h^2(\cot^2 12-\cot^2 16)=1600^2$

$h^2=\frac{1600^2}{\cot^2 12-\cot^2 16}$

$h>0$

$\therefore h=\sqrt{\frac{1600^2}{\cot^2 12-\cot^2 16}}$

$=506.69\text{m}$

20 (a) $\sin\theta-\sqrt{3}\cos\theta=0$

$\sin\theta=\sqrt{3}\cos\theta$

$\frac{\sin\theta}{\cos\theta}=\frac{\sqrt{3}\cos\theta}{\cos\theta}$

$\tan\theta=\sqrt{3}$

r.a.∠ $=\tan^{-1}(\sqrt{3})$

$=60°$

$\theta=60°,\ 180°+60°$

$\theta=60°,\ 240°$

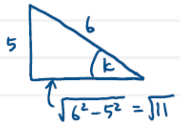
(b) $y = 3\sin x$

Goes through $(k, \frac{5}{2})$

$\Rightarrow$ At $x = k$, $y = \frac{5}{2}$

$\frac{5}{2} = 3\sin(k)$

$\sin(k) = \frac{5}{6}$



However $(k, \frac{5}{2})$ lies in between $\frac{\pi}{2}$ and π,

$\therefore$ 2nd quadrant

$\therefore \tan(k) = -\frac{5}{\sqrt{11}}$

(c) Turning point at $(0,0)$

$\Rightarrow f(0) = 0$

$\Rightarrow f'(0) = 0$

$0 = a + b(0) + c(0)^2 + d(0)^3$

$a = 0$

$f'(x) = b + 2cx + 3dx^2$

$0 = b + 2c(0) + 3d(0)^2$

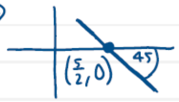
$b = 0$

$\therefore y = cx^2 + dx^3$

Slopes 45° downwards at $(\frac{5}{2}, 0)$

$\Rightarrow f(\frac{5}{2}) = 0$

$\Rightarrow$



$f'(\frac{5}{2}) = -1$

$0 = c(\frac{5}{2})^2 + d(\frac{5}{2})^3$

$0 = \frac{25}{4}c + \frac{125}{8}d$

$0 = 50c + 125d$ ①

$f'(x) = 2cx + 3dx^2$

$-1 = 2c(\frac{5}{2}) + 3d(\frac{5}{2})^2$

$-1 = 5c + \frac{75}{4}d$

$-4 = 20c + 75d$ ②

② × 2.5

$-10 = 50c + 187.5d$ ③

① − ③

$10 = -62.5d$

$d = -0.16 = -\frac{4}{25}$

Sub $d = -\frac{4}{25}$ into ①

$0 = 50c + 125(-\frac{4}{25})$

$0 = 50c - 20$

$c = \frac{2}{5}$

$\therefore y = \frac{2}{5}x^2 - \frac{4}{25}x^3$

or

$y = 0.4x^2 - 0.16x^3$