Barker College

2021 YEAR 11 PRELIMINARY EXAMINATION

Mathematics Advanced

Staff Involved:

- HYB: H Bawa
- GPF: G Fitzmaurice
- RAS: R Smith*
- PCB: P Boyages
- GDH: G Hanlon
- JZT: J Thomas
- AXD: A Davis
- KJL: K Lee
- ALY: A Young
- LMD: L de Gorter
- ESP: E Pratt

TOPICS COVERED:	
Algebra, Number and Surds	Exponential and Logarithmic Functions
Functions and Graphs	Differentiation and Curve Sketching
Transformations and Symmetry	Extending Calculus
The Coordinate Plane	Probability
Trigonometry	

205 copies — Friday 3 September, 8:30am

General Instructions:

- Reading Time – 5 minutes
- Working Time - 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- Reference sheet is provided separately and at the end of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for careless or poorly arranged working
- Diagrams are not to scale unless specifically stated

Total marks: 80

Section I – 10 marks (pages 2-5)

- Use Canvas to select your preferred response for Questions 1-10
- Allow about 15 minutes for this section

Section II – 70 marks (pages 6-14)

- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section
- Answer the questions on your own lined paper, clearly indicating which question you are answering
- Write your student number on all answer pages
- Section II will be uploaded in 4 sections: Q11, Q12, Q13, Q14

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use Canvas to select your preferred response for Questions 1 – 10.

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1. Which of the following is the value of $|5-7|+|-6|-|-5|+2|-3|$?

A. -9

B. -7

C. 9

D. 19

2. Which of the following is the expansion of $\left(5-2\sqrt{3}\right)^2$?

A. -11

B. $25-4\sqrt{3}$

C. 37

D. $37-20\sqrt{3}$

3. What is the *domain* of the function $y=\sqrt{5x-2}$?

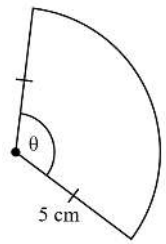
A. $[\,0\,,\infty)$

B. $[\,\frac{2}{5}\,,\infty)$

C. $(\,\frac{2}{5}\,,\infty)$

D. $(\,\infty\,,\,\frac{2}{5})$

4. The following sector has an area of 10π cm^2.
What is the size of the angle θ, subtended at its centre?



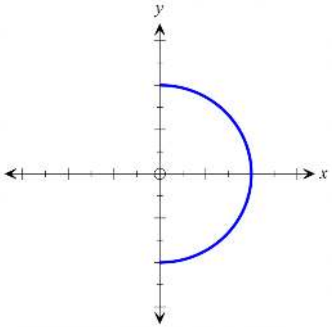
A. $\frac{\pi}{5}$

B. $\frac{2\pi}{5}$

C. $\frac{4\pi}{5}$

D. 2π

5. What type of relation is the following graph?



A. One-to-one

B. One-to-many

C. Many-to-one

D. Many-to-many

6. What is the angle of inclination, created with the x-axis, of the line $4x - 5y + 30 = 0$?

A. $38°40'$

B. $51°20'$

C. $75°58'$

D. $141°20'$

7. The function $y = x^2$ is translated 3 units to the right and 5 units up.
What is its new equation?

A. $y = x^2 + 3x + 5$

B. $y = x^2 - 6x + 14$

C. $y = x^2 - 9x + 5$

D. $y = x^2 + 9x + 14$

8. If $\log_x y = 2\log_x a + \log_x \left(\frac{b}{a}\right) - \log_x b$, then the value of y is:

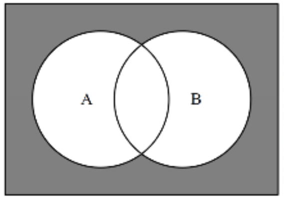
A. a

B. ab

C. a^2b

D. 1

9. A Venn diagram is shaded as per below.
Which of the following represents the shaded area in set notation?



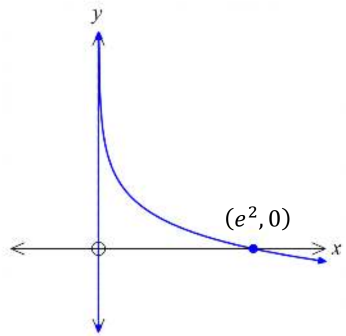
A. $A \cup B$

B. $A \cap B$

C. $\bar{A} \cup \bar{B}$

D. $\bar{A} \cap \bar{B}$

10. The graph of the function $y = a\ln x + b$ is shown below.
What are the values of *a* and *b*?



A. $a = 1, b = e^2$

B. $a = 1, b = 0$

C. $a = -1, b = 2$

D. $a = -1, b = e$

End of Section I

Section II

70 marks
Attempt Questions 11-14
Allow about 1 hour and 45 minutes for this section
Write your student number on all answer pages.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

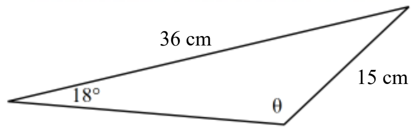
START A NEW PAGE SINCE QUESTION 11 WILL BE UPLOADED SEPARATELY

Question 11 (17 marks)

a. Convert 5 radians into degrees and minutes, rounded to the nearest minute. **2**

b. Solve $|4x - 9| = 15$. **2**

c. Find the value of the obtuse angle θ in the triangle below.
Round your answer to the nearest minute. **2**



d. If $f(x) = \frac{3}{x^2}$, find an expression for $f(f(x))$. **2**

e. Given $\log_x 4 \approx 1.26$ and $\log_x 6 \approx 1.63$, evaluate:

i. $\log_x 24$ **1**

ii. $\log_x 9$ **2**

f. The population of a small town is given by $P = 20\,000e^{kt}$, where *t* is the time in years after 1st January, 1900. After 5 years, the town's population had halved.

i. Find the initial population. **1**

ii. Calculate the value of the constant *k* to 4 decimal places. **2**

iii. What was the population in 1920? **1**

iv. In which year did the population fall below 100 residents? **2**

End of Question 11

START A NEW PAGE SINCE QUESTION 12 WILL BE UPLOADED SEPARATELY

Question 12 (17 marks)

a. During a particular month, the probability of it raining on a given day is $\frac{1}{3}$.
Over a three-day long weekend, what is the probability that it will rain on exactly two days? **2**

b. Differentiate the following. Leave your answers with a positive, non-fractional index, where necessary.

α. $y = 4e^{3x} + 6$ **1**

β. $y = (3x^2 + 5x)^4$ **1**

γ. $f(x) = 5x^{-2} - 6x^{\frac{1}{2}}$ **2**

δ. $f(x) = 3xe^{2x}$ **2**

c. Find the value of $\sin\frac{\pi}{3} + \sec\frac{\pi}{4}$.
Leave your answer as a single fraction in exact form. **2**

d. Find the value(s) of k for which $y = 3x^2 - 12x + k$ would have two different roots. **2**

e. Determine the transformation that has been applied to $x^2 + y^2 = 49$ to result in $x^2 + y^2 - 6y - 40 = 0$. **2**

f. Sketch the function $f(x) = 1 - \frac{1}{x-2}$, showing all intercepts and other important features. **3**

End of Question 12

START A NEW PAGE SINCE QUESTION 13 WILL BE UPLOADED SEPARATELY

Question 13 (18 marks)

a. Differentiate $f(x) = \frac{5x^3}{2x-1}$ **2**

b. Consider the curve $y = x^3 - px^2 + 2x + 5$. At the point where $x = 2$, the tangent to this curve is parallel to the line $y = -6x + 3$. Find the value of p. **3**

c. In a class of 25 students, 18 play netball, 13 play soccer and 2 play neither sport.

i. Draw a Venn diagram for this scenario, showing all information. **2**

ii. A student is chosen at random. Find the probability that:

α. They play both soccer and netball. **1**

β. They do not play soccer. **1**

d. Find the exact equation of the straight line which passes through the midpoint of (0, -5) and (4, -1) and is perpendicular to the line that makes an angle of 30º with the positive x-axis. **3**

e. For the function $y = -(x+2)(x-1)^2$:

i. Graph the function, showing x and y-intercepts. **2**

ii. Show very clearly, without skipping steps, why $\frac{dy}{dx} = 3(1-x)(x+1)$. **2**

iii. Find the coordinates of any stationary points and add them to your graph. **2**

Note that if these stationary points do not seem to fit your graph from **i.**, this indicates a mistake somewhere. As you try to resolve the mistake, be careful not to compound the error by altering anything that is correct.

End of Question 13

START A NEW PAGE SINCE QUESTION 14 WILL BE UPLOADED SEPARATELY

Question 14 (18 marks)

a. Three sets are defined as below.

Universal Set V = {Positive integers ≤ 20}
Subset A = {Even numbers}
Subset B = {Multiples of 3}   Note that 3 is a multiple of 3.

A number is randomly selected from Set V. Calculate:

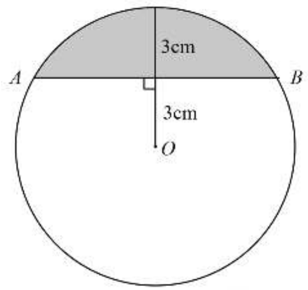
i. $P(A)$ **1**

ii. $P(B)$ **1**

iii. $P(A \cap B)$ **1**

iv. $P(A \cup B)$ **1**

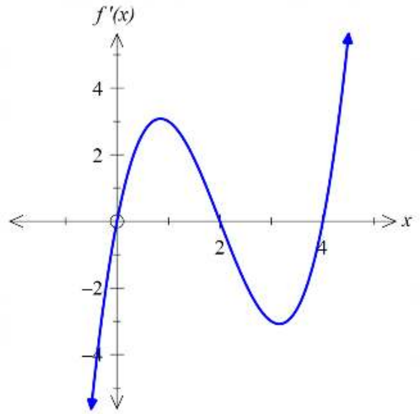
b. A circle has a radius of 6cm and centre O. A chord AB has been drawn, such that it bisects the radius, as shown below.



i. Show the size of $\angle AOB$ is $\frac{2\pi}{3}$ radians. **2**

ii. Calculate the area of the shaded minor segment to 2 decimal places. **3**

c. A function $y = f(x)$ has its **gradient function** $y = f'(x)$ graphed as below.



i. For which x value(s) does the original function $f(x)$ have a horizontal tangent? **1**

ii. For what values of x is the function $f(x)$ decreasing? **1**

d. Show that $\left(\operatorname{cosec} x + \frac{\cos x}{\sin x}\right)\left(\operatorname{cosec} x - \frac{\cos x}{\sin x}\right) = 1$. **2**

e. Let $f(x) = \ln x$, and $g(x) = x^4 + 1$.

i. Find $h(x)$ if $h(x) = f\big(g(x)\big)$. **1**

ii. State the domain and range of $h(x)$. **1**

iii. State whether the functions $f(x)$, $g(x)$ and $h(x)$ are even, odd or neither. **1**

iv. Show that $h(x) + h(-x) = f\left(\left(g(x)\right)^2\right)$. **1**

v. Let $i(x) = g(f(x))$.
By first determining whether or not $i(x) = 4ln x + 1$, state the range of $i(x)$. **1**

End of Paper

Year 11 Mathematics Advanced - Student Solutions

Section I: Multiple Choice

1. $|5-7|+|-6|-|-5|+2|-3|$

$= |-2| + 6 - 5 + 2\times 3$

$= 2 + 6 - 5 + 6$

$= 9$ **C**

2. $(5-2\sqrt{3})^2$

$=(5-2\sqrt{3})(5-2\sqrt{3})$

$=25-20\sqrt{3}+12$

$=37-20\sqrt{3}$ **D**

3. Domain of $y=\sqrt{5x-2}$

$5x-2\geq 0$

$x \geq \frac{2}{5}$

$[\frac{2}{5}, \infty)$ **B**

4. $A=\frac{1}{2}r^2\theta$

$10\pi = \frac{1}{2}\times 5^2 \times \theta$

$\theta = \frac{4\pi}{5}$ **C**

5. One-to-many **B**

6. $4x-5y+30=0$

$5y = 4x+30$

$y=\frac{4}{5}x+6$

$m=\frac{4}{5}$

$\frac{4}{5}=\tan\theta$

$\theta = \tan^{-1}(\frac{4}{5})$

$\theta = 38°40'$ **A**

7. $y=x^2 \Rightarrow y=(x-3)^2+5$

$y=(x-3)(x-3)+5$

$y=x^2-6x+9+5$

$y=x^2-6x+14$ **B**

8. $\log_x y = 2\log_x a + \log_x(\frac{b}{a}) - \log_x b$

$\log_x y = \log_x a^2 + \log_x(\frac{b}{a}) - \log_x b$

$\log_x y = \log_x(\frac{a^2 b}{a}) - \log_x b$

$\log_x y = \log_x(\frac{a^2 b}{ab})$

$\log_x y = \log_x a$

$\therefore y=a$ **A**

9. $\bar{A}\cap\bar{B}$ **D**

10. $a=-1$

so $y=-\ln x + b$

The curve passes through $(e^2, 0)$

$0=-\ln e^2 + b$

$0=-2\ln e + b$

$0=-2+b$

$b=2$ **C**

Section II: Question 11

a) $1 \text{ radian} = \frac{180°}{\pi}$

$5 \text{ radians} = 5\times\frac{180°}{\pi}$

$= 286°29'$

b) $|4x-9|=15$

$4x-9=15$ or $4x-9=-15$

$4x=24$ $\quad$ $4x=-6$

$x=6$ $\quad$ $x=-\frac{3}{2}$

c) $\frac{\sin\theta}{36} = \frac{\sin 18}{15}$

$\sin\theta = \frac{36\sin 18}{15}$

$\sin\theta = 0.7416\ldots$

$\theta = 47.87\ldots°, 180-47.87\ldots$

$\theta = 47.87\ldots, 132.1286\ldots$

since θ is obtuse

$\theta = 132°8'$

d) $f(f(x)) = \frac{3}{(\frac{3}{x^2})^2}$

$=\frac{3}{\frac{9}{x^4}}$

$=\frac{3x^4}{9}$

$=\frac{x^4}{3}$

e) i) $\log_x 24 = \log_x 4 + \log_x 6$

$\approx 1.26 + 1.63$

≈ 2.89

ii) $\log_x 9 = \log_x(\frac{36}{4})$

$=\log_x 36 - \log_x 4$

$=\log_x 6^2 - \log_x 4$

$=2\log_x 6 - \log_x 4$

$\approx 2\times 1.63 - 1.26$

≈ 2

f) i) $P=?$, when $t=0$

$P=20000e^{k\times 0}$

$P=20000e^0$

$P=20000$

ii) $P=10000$ when $t=5$

$10000=20000e^{5k}$

$\frac{10000}{20000}=e^{5k}$

$\frac{1}{2}=e^{5k}$

$\ln(\frac{1}{2})=5k$

$k=\frac{1}{5}\ln(\frac{1}{2})$

$k=-0.1386$

iii) $P=?$ $t=20$

$P=20000e^{-0.1386\times 20}$

$P=1250$

iv) $P=100$, $t=?$

$100=20000e^{-0.1386t}$

$\frac{100}{20000}=e^{-0.1386t}$

$\ln(\frac{100}{20000})=-0.1386t$

$t=\ln(\frac{100}{20000})\div -0.1386$

$t=38.2\ldots$

$\therefore$ The population fell below 100 residents in 1938.

Question 12

a) Probability of rain on two days

$P(RRR') = \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3}$

$= \frac{2}{27}$

or

$P(RR'R) = \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3}$

$= \frac{2}{27}$

or

$P(R'RR) = \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3}$

$= \frac{2}{27}$

$\therefore$ Probability of rain on exactly two days

$\frac{2}{27} + \frac{2}{27} + \frac{2}{27}$ or $3 \times \frac{2}{27}$

$= \frac{2}{9}$

b) α. $y = 4e^{3x} + 6$

$\frac{dy}{dx} = 3 \times 4e^{3x} + 0$

$\frac{dy}{dx} = 12e^{3x}$

β. $y = (3x^2+5x)^4$

$\frac{dy}{dx} = 4 \times (3x^2+5x)^{4-1} \times (6x+5)$

$\frac{dy}{dx} = 4(6x+5)(3x^2+5x)^3$

γ. $f(x) = 5x^{-2} - 6x^{\frac{1}{2}}$

$f'(x) = -2 \times 5x^{-3} - \frac{1}{2} \times 6x^{-\frac{1}{2}}$

$f'(x) = -10x^{-3} - 3x^{-\frac{1}{2}}$

$f(x) = -\frac{10}{x^3} - \frac{3}{\sqrt{x}}$

δ. $f(x) = 3xe^{2x}$

$u = 3x \quad v = e^{2x}$

$u' = 3 \quad v' = 2e^{2x}$

$f'(x) = 3 \times e^{2x} + 2e^{2x} \times 3x$

$f'(x) = 3e^{2x} + 6xe^{2x}$ or $3e^{2x}(1+2x)$

c) $\sin(\frac{\pi}{3}) + \sec(\frac{\pi}{4})$

$= \sin(\frac{\pi}{3}) + \frac{1}{\cos(\frac{\pi}{4})}$

$= \frac{\sqrt{3}}{2} + \frac{1}{\frac{1}{\sqrt{2}}}$

$= \frac{\sqrt{3}}{2} + \sqrt{2}$

$= \frac{\sqrt{3} + 2\sqrt{2}}{2}$

d) $y = 3x^2 - 12x + k$ has two different roots when $b^2 - 4ac > 0$

$(-12)^2 - 4(3)(k) > 0$

$144 - 12k > 0$

$144 > 12k$

$k < 12$

e) $x^2 + y^2 = 49 \rightarrow x^2 + y^2 - 6y - 40 = 0$

$x^2 + (y-3)^2 - 3^2 - 40 = 0$

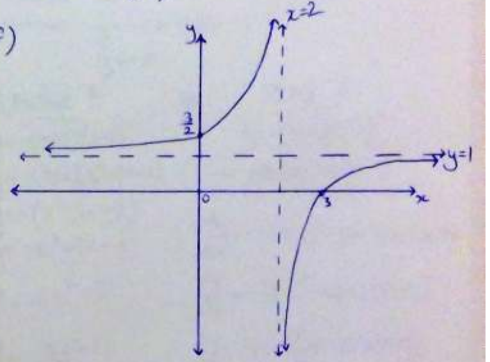
$x^2 + (y-3)^2 = 49$

Vertical translation of 3 units up

or

Translation $\binom{0}{3}$

f)



$f(x) = 1 - \frac{1}{x-2}$

Vertical asymptote: $x = 2$

Horizontal asymptote: $y = 1$

y-intercept: $f(0) = 1 - \frac{1}{0-2}$

$= 1 + \frac{1}{2}$

$= \frac{3}{2}$

x-intercept: $0 = 1 - \frac{1}{x-2}$

$\frac{1}{x-2} = 1$

$x - 2 = 1$

$x = 3$

Question 13

a) $f(x) = \frac{5x^3}{2x-1}$

$u = 5x^3 \quad v = 2x-1$

$u' = 15x^2 \quad v' = 2$

$v^2 = (2x-1)^2$

$f'(x) = \frac{15x^2(2x-1) - 2 \times 5x^3}{(2x-1)^2}$

$f'(x) = \frac{30x^3 - 15x^2 - 10x^3}{(2x-1)^2}$

$f'(x) = \frac{20x^3 - 15x^2}{(2x-1)^2}$ or $\frac{5x^2(4x-3)}{(2x-1)^2}$

b) $\frac{dy}{dx} = -6$ when $x = 2$

$y = x^3 - px^2 + 2x + 5$

$\frac{dy}{dx} = 3x^2 - 2px + 2$

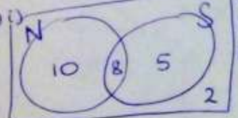
$-6 = 3(2)^2 - 2p \times 2 + 2$

$-6 = 12 - 4p + 2$

$4p = 20$

$p = 5$

c) i)



$25 - 2 = 23$

$|N \cup S| = 23$

$|N \cup S| = |N| + |S| - |N \cap S|$

$23 = 18 + 13 - |N \cap S|$

$|N \cap S| = 8$

ii) α. $\frac{8}{25}$ β. $\frac{12}{25}$

d) $m = \tan 30$

$m = \frac{1}{\sqrt{3}}$

$m_{perpendicular} = -\sqrt{3}$

Midpoint $\left(\frac{0+4}{2}, \frac{-5+-1}{2}\right)$

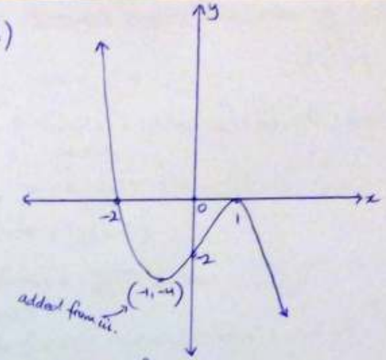
$(2, -3)$

$y - -3 = -\sqrt{3}(x-2)$

$y + 3 = -x\sqrt{3} + 2\sqrt{3}$

$y = -x\sqrt{3} + 2\sqrt{3} - 3$

e) i)



$y = -(x+2)(x-1)^2$

Shape: ↖↘

x-intercepts: -2, 1 ← bounce

y-intercept: $y = -(0+2)(0-1)^2$

$y = -2$

ii) Method A

$y = -(x+2)(x-1)^2$

$y = -(x+2)(x^2-2x+1)$

$y = -(x^3 - 3x + 2)$

$y = -x^3 + 3x - 2$

$\frac{dy}{dx} = -3x^2 + 3$

$\frac{dy}{dx} = -3(x^2-1)$

$\frac{dy}{dx} = -3(x+1)(x-1)$

$\frac{dy}{dx} = 3(1-x)(x+1)$

or

Method B

$y = -(x+2)(x-1)^2$

$u = -(x+2) \quad v = (x-1)^2$

$u' = -1 \quad v' = 2(x-1)$

$\frac{dy}{dx} = -1 \times (x-1)^2 + 2(x-1) \times -(x+2)$

$\frac{dy}{dx} = -(x-1)^2 - 2(x-1)(x+2)$

$\frac{dy}{dx} = -(x-1)[(x-1) + 2(x+2)]$

$\frac{dy}{dx} = -(x-1)(3x+3)$

$\frac{dy}{dx} = (1-x)(3x+3)$

$\frac{dy}{dx} = 3(1-x)(x+1)$

iii) $\frac{dy}{dx} = 3(1-x)(x+1)$

$0 = 3(1-x)(x+1)$

$x = 1$ or -1

when $x = 1$, $y = 0$ (from i.)

when $x = -1$, $y = -(-1+2)(-1-1)^2$

$y = -4$

$(1, 0)$ and $(-1, -4)$

Question 14

a) Method A: Listing elements of each set

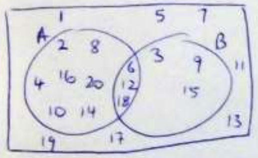
$A=\{2,4,6,8,10,12,14,16,18,20\}$

$B=\{3,6,9,12,15,18\}$

$A\cap B=\{6,12,18\}$

$A\cup B=\{2,3,4,6,8,9,10,12,14,15,16,18,20\}$

Method B: Venn Diagram

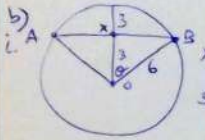


i. $\frac{10}{20}=\frac{1}{2}$ ii. $\frac{6}{20}=\frac{3}{10}$

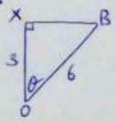
iii. $\frac{3}{20}$ or $P(A)\times P(B)$
$\frac{1}{2}\times\frac{3}{10}=\frac{3}{20}$

iv. $\frac{13}{20}$ or $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$P(A\cup B)=\frac{1}{2}+\frac{3}{10}-\frac{3}{20}$
$=\frac{13}{20}$

b) i.



Let X be the midpoint of AB



$\cos\theta=\frac{3}{6}$

$\theta=\cos^{-1}\left(\frac{1}{2}\right)$

$\theta=60°$ or $\frac{\pi}{3}$

$\therefore \angle AOB=2\times 60°$
$=120°$
$=120\times\frac{\pi}{180}$
$=\frac{2\pi}{3}$

or $2\times\frac{\pi}{3}$
$=\frac{2\pi}{3}$

ii. Area of minor segment = Area of minor sector − triangle

Area of minor sector

$A=\frac{1}{2}\times 6^2\times\frac{2\pi}{3}$

$A=18\pi\,cm^2$ (or 37.699...)

Area of triangle

$A=\frac{1}{2}\times 6\times 6\times\sin\left(\frac{2\pi}{3}\right)$

$A=9\sqrt{3}\,cm^2$ (or 15.588...)

$\therefore$ Area of minor segment

$18\pi-9\sqrt{3}$

$=22.11\,cm^2$

c) i. Horizontal tangents occur when $\frac{dy}{dx}=0$ or $f'(x)=0$

$\therefore x=0,2,4$

ii. A function is decreasing when the gradient ($f'(x)$) is negative.

$\therefore x<0,\ 2<x<4$

or

$(-\infty,0)\cup(2,4)$

d) $\left(\operatorname{cosec}x+\frac{\cos x}{\sin x}\right)\left(\operatorname{cosec}x-\frac{\cos x}{\sin x}\right)$

Method A

LHS $=(\operatorname{cosec}x+\cot x)(\operatorname{cosec}x-\cot x)$

$=\operatorname{cosec}^2x-\cot^2x$

$=1=$ RHS   (rearrangement of $1+\cot^2x=\operatorname{cosec}^2x$)

Method B

LHS $=\left(\frac{1}{\sin x}+\frac{\cos x}{\sin x}\right)\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)$

$=\left(\frac{1+\cos x}{\sin x}\right)\left(\frac{1-\cos x}{\sin x}\right)$

$=\frac{1-\cos^2x}{\sin^2x}$

$=\frac{\sin^2x}{\sin^2x}$

$=1$ $=$ RHS

e) i. $h(x)=f(g(x))$
$=\ln(x^4+1)$

ii. Domain

For $\ln A$, $A>0$.

Since $x^4+1\geq 1$, $\ln(x^4+1)$ is valid for all values of x

$\therefore x\in\mathbb{R}$, $(-\infty,\infty)$

Range

Since $x^4+1\geq 1$, the minimum value of $\ln(x^4+1)$ is $\ln 1=0$.

$\therefore h(x)\geq 0$ or $[0,\infty)$

iii. $f(x)\to$ neither $g(x)\to$ even $h(x)\to$ even

iv. Since $h(x)$ is even, $h(x)=h(-x)$

LHS $=h(x)+h(-x)$
$=h(x)+h(x)$
$=2h(x)$
$=2\ln(x^4+1)$
$=\ln(x^4+1)^2=\ln(g(x)^2)$
$=f(g(x)^2)=$ RHS

v. $i(x)=g(f(x))$
$=(\ln x)^4+1$

$(\ln x)^4+1\neq 4\ln x+1$

Since

$\ln(x^4)+1=4\ln x+1$

$(\ln x)^4\geq 0$

so $(\ln x)^4+1\geq 1$

Range: $i(x)\geq 1$

or

$[1,\infty)$