HSC Marking Feedback 2017

Mathematics Extension 1

Written Examination

Question 11

Part (a)

- The large majority of responses showed correct substitution into the formula $x = \frac{kx_2 + lx_1}{k+l}$ given on the Reference Sheet to achieve the correct result $x = -2$.
- In some responses the answer was unsimplified, for example $x = \frac{-10}{5}$, and others showed the y-coordinate as well.

Part (b)

- In most responses, students were able to differentiate $\tan^{-1}(x)$ correctly.
- A substantial number relied on the Reference Sheet formula $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ and were unable to identify the appropriate relationship between the formula and the x^3. The formula can be viewed as: $\int \frac{1}{a^2+\square^2}\,d\square = \frac{1}{a}\tan^{-1}\frac{\square}{a} + C$, using a $\square$ rather than an x.
- Common errors were algebraic, such as $(x^3)^2 = x$ or x^9.
- In a few responses, students changed the subject to x and then attempted to find $\frac{dx}{dy} = \frac{d}{dy}\left((\tan y)^{\frac{1}{3}}\right)$. If they correctly found this derivative they then had to change the variable back to x. Most were unsuccessful.
- In some responses, students incorrectly wrote $\tan^{-1}(x^3) = \frac{1}{\tan(x^3)}$.

Part (c)

- The inequality $\frac{2x}{x+1} > 1$ was solved using a number of different methods.
- In many responses students multiplied by the square of the denominator, others wrote $\frac{2x}{x+1} - 1 > 0$, or found critical values and tested.
- Some solved the equality, noting $x \neq -1$ and tested.
- The majority of response stated that 1 and – 1 were somehow significant and used these values.
- Some responses showed invalid conclusions such as: $1 < x < -1$.
- In many responses students wrote $x < -1$ and $x > 1$ or $x < -1, x > 1$ rather than the correct solution $x < -1$ or $x > 1$.

Part (d)

- Many responses correctly graphed $y = 2\cos^{-1} x$, showing or stating the correct domain and range. Others attempted to draw the graph $= \cos\left(\frac{y}{2}\right)$.
- In some responses, students confused the shape, included asymptotes or wrote incorrect domain and/or range.

Part (e)

- Students were asked to determine the value of $\int_0^3 \frac{x}{\sqrt{x+1}} dx$ given the substitution $x = u^2 - 1$.
- Those students who were able to differentiate implicitly to arrive at $dx = 2udu$ and change the bounding limits correctly were able to execute the correct solution. However, some students using this approach made errors substituting either the variables or the limits.
- Common errors were:
  - $x = u^2 + 1$, $dx = \frac{du}{2u}$ and $\frac{1}{u} \times 2u = u$
  - in some responses, students found the limits $u = \pm 2u$ and $u = \pm 1$ which led to dual solutions or confused the x with the u when calculating the limits.
- In many responses students changed the subject to u such that $u = \sqrt{x+1}$ or $u = \pm\sqrt{x+1}$, introducing roots and cases. From there, the possibility for errors increased.
- In other responses students found $\left[\frac{u^3}{3} - 2u\right]_1^2$ correctly but then confounded the fraction calculations.

Part (f)

- This part tested if the student could recognise that $\int \sin^2 x \cos x dx$ was of the form $\int (f(x))^n\ f'(x) dx$ and thus $\int (f(x))^n\ f'(x) dx = \frac{(f(x))^{n+1}}{n+1}$
- Approximately half of the responses demonstrated the use of mental strategies to obtain the answer to $\frac{\sin^3 x}{3}$. The other half used the $\cos 2\theta$ results and substituted to arrive at $\int (1 - \cos^2 x) \cos x dx$. Most then proceeded to integrate both factors together or to keep substituting.

Part (g)

- A number of students did not attempt any part of 11(g).

Part (g) (i)

- In many responses students were able to write down the correct expression. Some omitted the binomial coefficient ${}^8C_3$, that is, $\binom{8}{3}$or wrote ${}^8P_3$.
- In some responses the indices were swapped in error.

Part (g) (ii)

- This part was mostly answered correctly, even if (g) (i) was answered incorrectly.
- Omission of brackets was common, such as $\frac{4}{5}^8$ meaning $\left(\frac{4}{5}\right)^8$.

Part (g) (iii)

- The large majority of students demonstrated that the probability sum of complementary events is 1.
- Even if they were unable to answer (g) (ii) correctly, most answered this part correctly.
- In some responses the words *'at least one'* were confused with either *'one'* or *'more than one'*.

Question 12

Part (a)

- In better responses, the students found the reflex angle $\angle AOC = 260°$ before proceeding to the correct value.
- Alternate methods were common, the most popular being the approach which found a point on the circumference between A and C on the major arc AC and proceed from there.
- In both solutions, students needed to demonstrate that the angle at the centre is twice the angle at the circumference.
- Common errors were:
  - misquoting reasons, for example stating that the angle at the centre is half the angle at the circumference
  - assuming that the chords AB and BC were equal in length.

Part (b) (i)

- In the better responses, students used different colours to distinguish their two sketches and showed the sharp vertex point on both curves.
- Common errors were:
  - not showing a scale on their axes
  - not showing any intercepts even though they were asked
  - not using a ruler to draw the straight line segments
  - not realising the translations necessary for drawing both curves.

Part (b) (ii)

- The question required a correct approach in part (i).

- The better responses referenced the graphs and the points of intersection.
- Common errors were:
  - excluding the endpoints and writing $-1 < x < 2$
  - attempting to algebraically solve the equation
  - mistaking the word *'range'* in the question to mean finding a set of y values, not a 'range of values of x'.

Part (c) (i)

- In better responses students correctly equated the volumes in the given ratio 2:1 before simplifying their equation to show $3h^3 - 9h + 2 = 0$.
- Other solutions used the volume of the full sphere and equated the value of a single definite integral to a ratio of this full volume.
- Common errors were:
  - not equating the volumes in the correct ratio, interchanging the ratio as 1:2
  - omitting π from their definite integral and/or not squaring the function
  - difficulty with fractions and negative signs
  - incorrectly using areas and distances to attempt to find h.

Part (c) (ii)

- Common errors were:
  - mixing negative signs to arrive at $h_2 = -\frac{2}{9}$
  - incorrectly transcribing the formula from the reference sheet.

Part (d)

- In better responses, students found $\frac{dt}{dx}$ then $\frac{dx}{dt}$ first, then used the Reference sheet for the acceleration $\ddot{x} = \frac{d}{dx}(\frac{v^2}{2})$ to achieve the result.
- In the majority of responses, students made x the subject of the function and then proceeded to use the time equations. This worked well for students who substituted back to express the acceleration as a function of x.
- Common errors were:
  - not substituting back for $t = 4 - e^{-2x}$ in their acceleration/time function
  - not expressing $x = -\frac{1}{2}\ln(4 - t)$ correctly and thus not arriving at the correct solution
  - not being able to differentiate the above equation correctly
  - mistakenly thinking acceleration was $\frac{dv}{dx}$.

Part (e)

- In better responses, students used trigonometric substitution to gain the answer of 2.
- Common errors were:
  - mistakenly giving $\sqrt{2}$ as their answer
  - not using a double angle formula or choosing an inappropriate version
  - not knowing the fundamental limit $\lim_{x \to 0}(\frac{\sin x}{x}) = 1$.

Question 13

Part (a)

- In better responses, students recognised and used the formula $v^2 = n^2(a^2 - x^2)$ and correctly arrived at the answer by first establishing a pair of simultaneous equations. A good understanding of SHM was needed to know where to actually start the answer and go on to find n.
- Common errors were:
  - starting with $x = b + a\ \cos(nt + \alpha)$
  - starting with integrating $\ddot{x} = n^2(x - b)$.

Part (b) (i)

- A common error was ignoring the odd terms in the binomial expansions of $(1 + x)^n$ and $(1 - x)^n$.

Part (b) (ii)

- A common error was failing to multiply by -1 when applying the chain rule to find the derivative of $(1 - x)^n$.

Part (b) (iii)

- The responses that substituted $x = 1$ in part (ii) were generally successful.
- A common error was incorrect setting out of LHS and RHS.

Part (c) (i)

- In better responses, students used $y = 0$ to find the time t and substitute t into $x = vt\cos\theta$.
- A common error was using the Cartesian equation of the projectile and let $y = 0$ to show the horizontal range; this took too much time.

Part (c) (ii)

- A common error was assuming that the horizontal range is less than 100 m and showing $V^2 < 100g$.
- Some students were not sure how to use the fact that $\sin 2\theta \leq 1$.

Part (c) (iii)

- In better responses, students used a graphical approach.
- A common error was incorrectly justifying $\frac{\pi}{12} \leq \theta \leq \frac{5\pi}{12}$ from $\sin 2\theta \geq \frac{1}{2}$.

Part (c) (iv)

- In better responses students solved $\frac{dy}{dt} = 0$ for t and used it to establish a correct expression for maximum height.
- Common errors were:
  - leaving out $\sin\theta$ (1st term) in the expression of $y = V\left(\frac{Vsin\theta}{g}\right)\sin\theta - \frac{1}{2}g\left(\frac{Vsin\theta}{g}\right)^2$
  - using $\theta = \frac{\pi}{4}$ instead of $\theta = \frac{5\pi}{12}$ to establish the maximum.

Question 14

Part (a)

- Common errors were:
  - poor use of algebraic techniques including factorisation
  - simple arithmetical errors
  - not carrying over the letter used to indicate the multiple of 7; ie students would forget the P in $64P$ or $36P$, where P represented an integer.
- Many students gained marks by being able to set up and use a correct assumption. However, a significant number of students were unable to execute the proof because of difficulty handling/manipulating the powers of 8 and 6.
- In better responses, students who used a rearrangement of their assumption also drew attention to this rearrangement (eg 'using the assumption').

Part (b) (i)

- In better responses, students used the most straightforward method, which was to substitute the equation of the tangent, which was easily found using the Reference Sheet, and eliminate the y variable by substituting into the equation of the parabola.
- Many students spent time and effort deriving the equation of a tangent to a parabola. Having said that, the stated general result on the Reference sheet had to be adapted to the specific result for the given circumstances (ie $t = p$ and $a = 1$). This was often missed.
- A common error was carelessness with signs.
- A rarer, and less successful, method involved finding the coordinates of Q and R and then using the given equation; substituted into the equation and show that the coordinates 'satisfied' it.

Part (b) (ii)

- In better responses, students used the sum of the roots of the equation given in part (i) for the x-coordinate then substituted into the tangent to find the y-coordinate.
- Other responses involved:
  - actually finding the x-coordinates of P and Q using the quadratic formula, then using the mid-point formula
  - finding the axis of symmetry of the parabola
  - locating the stationary point of the quadratic.
- Much time was wasted attempting to find the y-coordinates of P and Q, and then applying the mid-point formula once more.
- A variety of methods were used to find the y-coordinate of M:
  - using $y = -\frac{x^2}{4a}$ and then the mid-point formula
  - using $y = px - p^2$ and then the mid-point formula, not realising they could have simply substituted the x-coordinate of M into the same equation
  - an elegant solution involved using $y_M = \frac{\frac{x_P^2}{-4a} + \frac{x_Q^2}{-4a}}{2}$; and then the relationship between the roots and coefficients.
- The most efficient solutions achieved the result in 3 or 4 lines.

Part (b) (iii)

- This part involved eliminating the parameter from the given mid-point and solving the resulting equation for a. However, many students did not realise that, as the equation of the locus had been given to them, all that was required was to substitute the given coordinates into the given equation.

- Of those students who did manage to arrive at the correct quadratic $a^2 = 2a + 1$, a number could not correctly solve this equation, often transforming the equation, incorrectly, to $(a - 1)^2 = 0$. Of those who correctly solved the equation and arrived at $a = 1 \pm \sqrt{2}$, a significant number did not go on to impose the condition that $a > 0$.

Part (c) (i)

- In better responses, students took note of the instruction 'by differentiating the product', correctly went on to substitute the given expression for $F'(t)$, and recognised the simple cancellation that was then needed.

- A common error was not using the product rule, instead making a statement such as $\frac{d}{dt}[F(t)e^{0.4t}] = 0.4F(t)e^{0.4t}$.

Part (c) (ii)

- Many responses ignored the integration of the LHS.

- Most students who attempted this question went on to use the result from (c) (i) to successfully show the given function.

Part (c) (iii)

- The most common errors were related to division of terms with base e, converting exponentials to logarithms and using basic log laws for example going from $\frac{5}{4} = \frac{e^{-0.4t}}{e^{-0.5t}}$ to $\ln\left(\frac{5}{4}\right) = \frac{-0.4t}{-0.5t}$.