

## 2016 Notes from the Marking Centre – Mathematics Extension 1

### Question 11

- (a) The question was generally done very well by a large number of candidates. Common problems were:
- making transcription errors
  - incorrectly giving the answer as  $y = \pm\sqrt[3]{x+2}$
  - writing  $y^3 = x - 2$  after correctly interchanging  $x$  and  $y$  and writing  $x = y^3 - 2$
- (b) The question was attempted well by the vast majority of candidates, with the rules for finding primitives of fractional powers of  $u$  handled correctly. Common problems were:
- not substituting back to give a primitive in terms of  $x$
  - incorrectly arranging the substitution to obtain  $x = u - 4$
- (c) In many responses, candidates attempted to use the Reference Sheet but struggled to fit the information into the appropriate standard integral, namely,  $\int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$ , often not handling the value of  $a$  to give the correct solution.

In many of the better responses, the function was rearranged to make  $x$  the subject, then  $\frac{dx}{dy}$  was found before finding  $\frac{dy}{dx}$  in terms of  $x$ . Common problems were:

- obtaining  $a = \frac{1}{3}$  from  $\frac{1}{a} = 3$  but then having an incorrect function of  $x$  having an incorrect constant multiple of the derivative.
- (d) This question was generally done very well, indicating sound knowledge of the fundamental limit. Common problems were:
- incorrectly manipulating the fractional expression
  - incorrect reasoning, simply cancelling all parts of the fraction to achieve  $\frac{2}{3}$
  - incorrectly manipulating the algebraic fraction, yielding the incorrect value of  $\frac{3}{2}$
- (e) Such inequations usually lend themselves to a wide range of solution types. A significant feature of the responses was not so much the method at arriving at the critical  $x$ -values, but analysing the correct section of the number line to give the correct solution. The question was challenging as there were 3 boundary values.

In the better responses, the approach was to first observe that  $x \neq -\frac{5}{2}$ , then to find two other boundary values,  $x = \frac{1}{2}$  and  $x = -3$ , and then finally to solve the inequation on the number line to yield  $x < -3$  or  $-\frac{5}{2} < x < \frac{1}{2}$ . Common problems were:

- making many varied transcription errors in writing out the question
- factorising the quadratic  $2x^2 + 5x - 3 = 0$  **incorrectly**
- when using the method beginning with multiplying through by  $(2x + 5)^2$ , not being able to arrive at the correct factorised cubic, typically expanding the line  $3(2x + 5) - x(2x + 5)^2 > 0$  rather than factorising it
- leaving the negative values off the boundary values when stating the solutions, in particular the boundary value  $-3$

(f)(i) This question was done well by most candidates with the better responses having the answer expressed

as  ${}^3C_1 \left(\frac{3}{5}\right)^1 \left(\frac{2}{5}\right)^2 = \frac{36}{125}$ . Common problems were:

- not realising there were 3 combinations of 1 Hit and 2 Misses, incorrectly giving  $\frac{12}{125}$  as the answer.

(f)(ii) Many candidates answered this question well, with the better solutions making use of complementary events, ie,  $P(\text{at least 2 Hits out of 6 throws}) = 1 - P(1\text{Hit or } 0\text{ Hits})$ .

Some responses indicated that candidates thought that the number of darts thrown was 3 thinking parts (i) and (ii) were connected. Common problems were:

- not realising there were  ${}^6C_1$  combinations of 1 Hit and 5 Misses, typically counting just 1 combination of same
- incorrectly thinking that  $P(\text{at least 2 Hits out of 6 throws}) = 1 - P(2\text{ Hits})$ , or  $P(\text{at least 2 Hits out of 6 throws}) = 1 - P(1\text{ Hit})$ .

## Question 12

(a)(i) In the large majority of responses, candidates wrote a proportion statement like  $\frac{r}{5} = \frac{h}{20}$  and arrived at the result. Some candidates gave a full similarity proof which is more than is required for one mark.

Others made disconnected statements like  $20 \div 5 = 4 \therefore r = \frac{h}{4}$ . Common problems were:

- not using mathematical concepts to justify the result required by the instruction 'explain' (or 'show')

(a)(ii) In many responses, candidates could substitute h into the given equation to get

$v = \frac{\pi h^3}{48}$  then differentiate with respect to h and arrive at the required result. Common problems were:

- differentiating and dealing with different variables, eg, differentiating (incorrectly)  $v = \frac{1}{3}\pi r^2 h$  with respect to h to gain  $\frac{dv}{dh} = \frac{1}{3}\pi r^2$ , or with respect to r to gain  $\frac{dv}{dr} = \frac{2}{3}\pi r h$ , and then arriving at the required result

- unnecessarily using the quotient rule to differentiate  $v = \frac{\pi h^3}{48}$

(a)(iii) In a substantial number of responses, candidates deduced that  $\frac{dA}{dh} = \frac{\pi h}{8}$  and found a relationship

between  $\frac{dh}{dt}$  and  $\frac{dA}{dt}$  leading to an efficient two-line solution. Common problems were:

- using completely invalid logic
- poor understanding of the chain rule, often leading to 'fudging' results

(a)(iv) Many responses included a valid proof using  $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ .

However, many responses appeared to have been worked starting with the given result  $\frac{dh}{dt} = \frac{-0.32}{\pi h}$ , eg,

$\frac{dh}{dt} = \frac{16}{\pi h^2} \times \frac{\square}{\square} = -\frac{0.32}{\pi h}$  so  $\frac{\square}{\square} = -\frac{0.02h}{1}$ , therefore  $\frac{dV}{dt} = -0.02h$ . Common problems were:

- using a variety of derivatives relating the variables in the question  $v$ ,  $h$ ,  $A$  (or  $S$ ),  $r$  and  $t$  and putting them into chain rules hoping one would work
- making substitution and arithmetic errors

(b)(i) This part relied on being able to interpret the sentence ‘the rate at which ...  $x$  is increasing is proportional to ...  $y$ ’ which proved to be challenging. Common problems were:

- not correctly interpreting the sentence as  $\frac{dx}{dt} = ky$
- quoting a familiar equation such as  $\frac{dx}{dt} = k(B - P)$ , hoping that it would lead somewhere and nominating values for the pronumerals

(b)(ii) This part was done relatively well. In most responses candidates could differentiate  $x = 500 - Ae^{-0.004t}$

and then rearrange in order to arrive at:  $\frac{dx}{dt} = 0.004(500 - x)$ . Common problems were:

- neglecting to show the rearrangement and making a statement such as ‘because it satisfies the equation’
- not showing all relevant steps, no matter how trivial they appear
- making transcription errors between 0.04 and 0.004, resulting in marred proofs

(c)(i) In many responses, candidates used the Reference Sheet to find the derivatives of  $\cos x$  and  $\tan x$ , then state that for perpendicular lines  $m_1 \times m_2 = -1$ .

Many candidates were able to see that if  $\tan \alpha = \cos \alpha$  then  $\frac{1}{\cos^2 \alpha} \times -\sin \alpha = -1$ .

In a reasonable number of responses, candidates solved  $\tan \alpha = \cos \alpha$  and showed  $\alpha = \sin^{-1}\left(\frac{-1 + \sqrt{5}}{2}\right)$

then went on to prove  $\frac{1}{\cos^2 \alpha} \times -\sin \alpha = -1$ .

In a few cases the quotient rule, which was very inefficient, was used to find the derivative of  $\tan x$ . Common problems were:

- fudging results, being unable to justify the conclusion

(c)(ii) In a reasonable number of responses, candidates were able to create the function to use from the equation  $\tan \alpha = \cos \alpha$  (eg,  $f(x) = \tan x - \cos x$ ), then go on to differentiate and substitute correctly. In a significant number of responses, candidates manipulated the equation first so  $f(x) = \cos^2 x - \sin x$ ,

$f(x) = \sin^2 x + \sin x - 1$  or  $f(x) = \frac{\tan x}{\cos x} - 1$  were common. These then had different approximations.

Common problems were:

- poor calculator skills. In some responses, ‘degree’ mode was used instead of radians.
- incorrectly choosing  $f(x) = \tan x$  or  $f(x) = \cos x$  which trivialised the solution substantially and suggests that the use of Newton’s method is not fully understood.

## Question 13

- (a)(i) In a good proportion of responses, candidates identified the amplitude or centre of motion with a graph or a calculation but did not show a calculation to connect  $n = \frac{4\pi}{25}$  to the model.

In a small number of responses, candidates chose to differentiate the given function twice and show that the resulting acceleration function satisfied the conditions for SHM. Common problems were:

- confusion over the difference between frequency and period
- the instruction 'Explain why' was not always understood. An explanation of how the numbers in the given function could be obtained from the given conditions was required, and simply listing the given conditions is not necessarily considered to be an explanation.

- (a)(ii) The most common approach to this question was via the use of calculus, although the inability to 'handle' the  $\pi$  when differentiating was evident.

The most successful approach was to solve  $\dot{x} = \frac{16\pi}{25}$  which generally led to the correct result.

Solving  $x = 5$  or  $\ddot{x} = 0$  usually led to finding the time when the tide was 'decreasing' at the fastest rate.

A less common, but very elegant solution was provided by candidates used their knowledge of the symmetry of the cosine function. These simply found  $\frac{3}{4}$  of the period and then added it to the starting time.

- (b)(i) This question was generally answered well.

Using  $y$  to find the time for greatest height was overwhelmingly the best and most used method.

A less common but equally successful method was to find the time of flight and concluding that the time to reach the greatest height is half the time of the flight.

Once the time was found, the given result was obtained by substituting into the given vertical displacement formula. Common problems were:

- difficulties with the algebraic manipulation involved when trying to find the Cartesian equation for the path of flight after eliminating  $t$

- (b)(ii) Those who recognised the connection to part (i) usually went on to gain full marks. Common problems were:

- failure to realise the benefit of deriving a general solution in part (b) (i); instead choosing to redo what was essentially the same question as part (b) (i). (The only difference being that part (b) (ii) uses specific conditions whereas part (b) (i) uses general conditions).

In many responses, candidates demonstrated good calculus skills by starting from scratch although considerable time would have been spent on working that was unnecessary. Common problems were:

- an inability to solve the simple equation  $5t^2 = \frac{125}{4}$
- failure to realise that this problem involved a new set of conditions; instead the conditions from part (b) (ii) were used, making it impossible to obtain a correct result
- the use of the equations of motion found in the Physics course rather than deriving such equations from first principles

- (b)(iv) This part was done quite well

- (c)(i) In most responses, candidates were able to successfully explain why  $AD$  was perpendicular to  $BM$ , most commonly via the angle in a semicircle is a right angle, and were able to demonstrate knowledge about the condition for a quadrilateral to be cyclic. Common problems were:

- not being able to put all relevant information into a coherent proof
- poor reasoning or even in some cases a lack of reasoning

- (c)(ii) This part was not done well and often not attempted. Common problems were:
- the inability to use correct logic and support the response with valid reasoning

### Question 14

- (a)(i) The most common successful methods included expanding the RHS and gathering like terms to arrive at LHS, and using the Remainder/Factor Theorem followed by long division.
- (a)(ii) In the majority of responses the initial case was verified and the inductive assumption was correctly applied. Common problems were:
- poor algebra skills
  - confusion between LHS & RHS expansions
- (b)(i) This was answered quite well in most cases.
- (b)(ii) Common problems were:
- the differentiation of  $(1+x)^n$ .
  - Stating that  $\frac{d2^n}{dx} = n2^n$ , instead of  $\frac{d(1+x)^n}{dx} = n(1+x)^{n-1}$ .
- (b)(iii) In most cases this was not attempted or the responses were quite poor. Common problems were:
- expressing it as a deductive proof
  - stating the connection between the expansions in parts (i) and (ii).
- (c)(i) The question was answered well with many correct solutions. Common problems were:
- failure to recognise that the equation of the tangent was given in the Reference Sheet, so time was spent deriving the tangent equation from first principles.
- (c)(ii) Common problems were:
- failure to recognise that the equation of the normal was given in the Reference Sheet and instead it was derived
  - while in many responses candidates were successful in finding the co-ordinates of R, they
    - were unable to find a way to successfully eliminate the parameter and go on to establish the locus, or,
    - found it difficult to sustain the level of algebra which was required in this question.
- (c)(iii) Responses here were largely dependent on the previous part (ii). Most candidates who got part (ii) correct were able to give the correct focal length.
- (c)(iv) In most cases, responses to this question were quite poor or it not attempted. Common problems were:
- not adequately expressing the link between gradients in terms of  $t$ ,
  - often making some fundamental arithmetic/algebraic error.