Mathematics

HSC Marking Feedback 2018

Question 11

Skills addressed:

- correctly multiply by $\frac{3-\sqrt{2}}{3-\sqrt{2}}$ (a)
- using the difference of 2 squares to expand the denominator (a)
- understanding the difference between a linear inequality and an absolute value inequality (b)
- understanding the need to reverse the inequality sign when dividing by a negative value (b)
- stating the two required equations and solving them simultaneously to find the common difference (di)
- showing the correct substitution into the formula for the n^{th} term of an arithmetic series, evaluating a and finding the 50$^{\text{th}}$ term (dii)
- finding the primitive and correctly evaluating the definite integral (e)
- listing expressions for u, u', v and v' and showing the correct substitution into the product rule (f)
- listing expressions for u, u', v and v' and showing the correct substitution into the quotient rule (g).

Areas for students to improve include:

- understanding why multiplying by a conjugate leads to a rational denominator (a)
- expanding binomial products (a)
- factorising before simplifying an algebraic fraction (c)
- using the Reference Sheet to obtain the rule for factorising the difference of 2 cubes (c)
- understanding that twentieth is 20$^{\text{th}}$ not 12$^{\text{th}}$ (di)
- using the Reference Sheet to obtain the formula for the n^{th} term of an arithmetic series (di)
- understanding the difference between an arithmetic and a geometric series (di)
- selecting the appropriate formula to use in series questions (dii)
- understanding the rules for differentiating and integrating exponential functions (e)
- using the correct order when substituting limits, that is, $F(3) - F(0)$ instead of $F(0) - F(3)$ (e)
- evaluating e^0 correctly (e)

- using the Reference Sheet to obtain the product rule and the rules for derivatives (f)
- using the Reference Sheet to obtain the quotient rule and the rules for derivatives (g)
- writing the function correctly when expressing the quotient as a product (g).

Question 12

Skills addressed:

- having a clear understanding and practical use of bearings and related notation (ai)
- presenting a correct equation noting the sum of two angles (ai)
- having a sound understanding of the use of both the sine and cosine rules (aii)
- quoting the correct formula with correct substitution (aii)
- ensuring final answer is provided as directed by the question (aii)
- copying the given diagram onto their answer sheet with correct labelling (ci)
- considering the linking of the various parts of a question (cii)
- carefully examining the question, considering if there is more than one approach and selecting the most appropriate and simplest method (cii)
- having a clear understanding of the difference between differentiation and integration and their use (di)
- having a clear understanding of the meaning of the term 'stationary' (dii).

Areas for students to improve include:

- being able to find the supplements and complements of given angles (ai)
- ensuring that they answer the given question as some students found the area of a triangle (ai)
- copying the correct formula from the Reference Sheet and then substituting the given values (aii)
- correctly following the order of operations in calculations (aii)
- completing all steps required as some students left their answer as AC^2 (aii)
- using the correct rule from the Reference Sheet to differentiate the trigonometric function (b)
- finding exact values of trigonometric expressions (b)
- providing a clear and sequential congruence proof with reasoning at each step (ci)
- using the properties of a square to equate sides (ci)
- correctly labelling pairs of angles and sides in their proof (ci)
- presenting correct and relevant reasoning for each step of their proof (ci)
- understanding and using congruence tests (ci)
- achieving the answer by subtracting twice the area of the triangle from the square (cii)
- correctly using the area formulae for each of the two relevant shapes (cii)
- understanding the definition and use of a perpendicular height (cii)
- linking the result found in (c i) to support the answer of (cii)

- showing accuracy when substituting and evaluating expressions (di)
- solving quadratic equations (dii)
- ensuring that they fully answer the question as many students only found the time when the acceleration was zero (diii)
- accurately substituting and evaluating expressions (diii).

Question 13

Skills addressed:

- showing numerical results in their testing of stationary points using either the first or second derivative (ai)
- using tables with labelled rows, indicating x , $\frac{dy}{dx}$ and slope, when using the first derivative test (ai)
- using a table of values to show the change in sign of the second derivative and hence a change in concavity (aii)
- indicating that they were substituting into the second derivative (aii)
- showing numerical results to verify their test (aii)
- stating a conclusion after their test (aii)
- factorising fully before solving $\frac{dy}{dx} = 0$ (ai)
- showing their numerical results when completing the testing of points (ai)
- stating conclusions by classifying their stationary points (ai)
- drawing a smooth curve of at least $\frac{1}{3}$ of a page, clearly showing important features (aiii)
- labelling all important features, including intercepts (aiii)
- labelling the axes (aiii)
- drawing the diagram given in the question onto their answer sheet (bi)
- annotating their diagram with information provided in the question and geometric facts used in their proof (bi)
- using correct geometric reasons (bi)
- using an appropriate test for similarity and stated a conclusion (bi)
- clearly showing the pairs of equal matching angles (bi)
- drawing the triangles separately to help determine matching sides (bii)
- clearly showing the ratio of matching sides, for example, $\frac{CB}{AB} = \frac{BD}{BC}$, before substituting the lengths given in the question (bii)
- correctly calculating the length of BD then AD (bii)
- showing the substitution of $P(t) = 184$ and $t = 50$ into $P(t) = 92e^{kt}$ (ci)
- using logarithms to solve for k (ci)
- understanding that the variable for population was given 'in millions' (ci)
- showing the substitution of $k = 0.0139$ and $t = 110$ into $P(t) = 92e^{kt}$(cii)

- calculating the population using a calculator (cii).

Areas for students to improve include:

- remembering that the second derivative is used to test concavity not slope (aii)
- realising that the y-coordinate is given in the question, so there is no need to show it (aii)
- understanding the difference between a point of inflexion and a horizontal point of inflexion (aiii)
- sketching the curve beyond the domain of the features to be shown, including passing through the x-intercept of 6 (aiii)
- understanding the different tests for similar triangles (bi)
- stating matching angle pairs correctly, for example, $\angle ACB = \angle CDB$ (bi)
- setting out a proof, providing statements, in a logical order stating reasons at each step (bi)
- defining variables used that are not given in the question (bii)
- understanding that if using the SAS test for similarity that the included angle is required (bii)
- recalculating the value for k if their result does not match value given in question (ci)
- if using 184 000 000 also use 92 000 000 (ci)
- understanding that t is the number of years after 1910, so 2020 is represented by $t = 110$ (cii).

Question 14

Skills addressed:

- showing the appropriate substitution into the area of a triangle formula,
  - that is, $A = \frac{1}{2} \times 3 \times 6 \times \sin 60°$ (ai)
- substituting the exact value of $\sin 60°$ into the formula before simplification (ai)
- presenting a simplified exact value as their final answer (ai)
- understanding that 'hence' implies linking parts (ai) and (aii)
- carefully examining a question, considering if there is more than one approach, and selecting the most appropriate and simplest method (aii)
- recognising that their result in (ai) is equal to the sum of the areas of the two smaller triangles (aii)
- stating that $V = \pi \int_1^{10} (y-1)^{\frac{1}{2}} dy$ (b)
- integrating and substituting limits correctly and presenting all intermediary steps (b)
- obtaining $f'(x)$, the discriminant of the derived function and knowing to use $\Delta < 0$ (c)
- correctly expressing their solution to the quadratic inequality (c)
- recognising the need to find the sum of an arithmetic and a geometric sequence (dii)
- correctly substituting into series formulas (dii)

- drawing a clearly labelled tree diagram (e)
- having a clear understanding of when to use addition or multiplication of fractions in probability (e)
- understanding the meaning of *'*at least one*'* in probability questions (e)
- using $P(\text{at least one pen faulty}) = 1 - P(\text{both pens not faulty})$ (ei)
- clearly presenting and using a tree diagram to establish the two correct options (eii)
- understanding that selecting machine A or machine B requires multiplying options by $\frac{1}{2}$. (eii).

Areas for students to improve include:

- assuming information not stated in the question without proof, for example, $\angle LKN = 90°$ (ai)
- expressing an answer in exact form (aii)
- understanding that rotating a region about the y-axis requires the formula $V = \pi \int_a^b x^2 \, dy$, where a and b are y-values (b)
- using algebraic skills to find an expression for x^2 in terms of y (b)
- integrating using the reverse chain rule (b)
- choosing and applying the correct limits (b)
- understanding of the relevance of the discriminant (c)
- understanding how to find the discriminant (c)
- solving quadratic inequalities (c)
- understanding the difference between T_n and S_n (di)
- understanding that 'on each of the first 3 days' implies three terms are needed (di)
- substituting values into a given formula prior to obtaining the final answer (di)
- recognising different types of sequences (dii)
- correctly obtaining the first term of either the arithmetic or geometric sequence (dii)
- using arithmetic and geometric series formulae (dii)
- accuracy of calculations when systematically adding all 20 terms (dii)
- being accurate in calculations (ei)
- setting up a probability tree diagram (ei)
- understanding when to reduce fractions if an event is repeated (eii)
- correctly using a probability tree diagram (eii).

Question 15

Skills addressed:

- showing the substitution of $t = 0$ into $L(t) = 12 + 2\cos(\frac{2\pi t}{366})$ (ai)
- using the fact that $-1 \leq \cos x \leq 1$ to find the range of $L(t)$ and hence the solution (aii)
- drawing a graph to find the minimum value (aii)
- understanding how to work with trigonometric functions in terms of π (aiii)
- equating the two parts of equal area using definite integrals, finding primitive functions and solving the resulting logarithmic equation (b)
- knowing that $k > 0$ and discarding $k = -15$ (b)
- knowing the required area could be represented by $A = \int_a^b \big(f(x) - g(x)\big)dx$ where a and b are x-values (ci)
- simplifying $f(x) - g(x)$ before finding the primitive function and calculating the definite integral (ci)
- using the correct formula for Simpson's rule and understanding that three function values are required for one application of the rule (cii)
- using their simplified function from (ci) to calculate their function values (cii)
- using a table to show their three functions values (cii)
- equating the gradient function for the tangent at P and the gradient of the line $y = 2x$ and solving for x (ciii)
- finding the y-coordinate by substitution (ciii)
- understanding that the triangle is not right angled (civ)
- using the formula for perpendicular distance provided in the Reference Sheet (civ).

Areas for students to improve include:

- knowing that $\cos 0 = 1$ (ai)
- using the mark allocation as a guide to the amount of working expected (aii)
- understanding how to find 'angles of any magnitude' (aiii)
- changing the subject of the equation to t (aiii)
- understanding the difference between radians and degrees when solving trigonometric equations (aiii)
- using the correct order when substituting limits into the primitive function (b)
- solving logarithmic equations (b)
- writing correct statements, involving definite integrals, to use in area problems (ci)
- applying absolute value of functions correctly (ci)
- using the formula from the reference sheet and understanding the meaning of $\frac{b-a}{6}$ (cii)
- finding the correct function to use in Simpson's rule (cii)
- showing substitutions used to find the y-coordinate (ciii)

- performing calculations with surds (ciii)
- understanding that the equation of the line used in the perpendicular distance formula needs to be in general form (civ)
- finding the correct angle at the origin to use in the sine rule (civ).

Question 16

Skills addressed:

- having a clear understanding of the steps required to 'show' a result (ai)
- using Pythagoras' theorem to correctly link the two given variables (ai)
- showing their substitution of information into the given volume formula (ai)
- listing expressions for u, u', v and v' and showing the correct substitution into the product rule (aii)
- simplifying their expression for $\frac{dV}{dx}$ showing setting out in logical steps (aii)
- understanding the process used for finding a maximum or minimum value (aiii)
- ensuring that they demonstrate a test to distinguish between minimum and maximum results (aiii)
- selecting a method to ensure all possible elements in the event and sample space are counted systematically (bi)
- presenting a table which identifies all options for each possible combination (bi)
- using the counting system established in part (bi) to support the solution in (bii)
- grouping options and set work out in a methodical manner (bii)
- having a clear understanding of the steps required to 'show' a result (c)
- presenting all work clear sequential steps (c)
- providing a detailed progression from A_1 through to A_2. (ci)
- responding to the direction 'show that' and providing a detailed progression from A_2 to A_3 (cii)
- being able to achieve an expression for A_n (cii)
- using the sum of a geometric progression formula to arrive at the given result (ciii)
- demonstrating a high degree of accuracy and skill in algebraic manipulation (ciii).

Areas for students to improve include:

- writing a correct statement for Pythagoras' theorem and rearranging the resulting equation (ai)
- using the pronumerals given in the question (ai)
- differentiating functions requiring the use of the chain rule (aii)
- manipulating algebraic fractions and leaving the result in simplified form (aii)
- omitting solutions for $\frac{dV}{dx} = 0$ which are invalid, that is, $x = 0$ (aiii)
- showing numerical results when completing the test for the maximum or minimum

value (aiii)

- using their x-value to find θ as required (aiii)
- developing a range of counting techniques to address the variety of stimulus found in probability questions (bi)
- understanding to have 'no chance of winning before rolling the third die', a double or consecutive number needed to be thrown (bi)
- recognising all possibilities available to achieve a win (bii)
- realising that the idea of a complement was not required (bii)
- using brackets correctly (ci)
- remembering to increase and subtract the withdrawal, that is, using $A2 = A1(1.04) - P(1.05)$ (ci)
- knowing that the third withdrawal was $P(1.05)^2$ and using $A_3 = A_2(1.04) - P(1.05)^2$ (cii)
- using patterns to obtain an expression for A_n (ciii)
- using the correct values for the first term and common ratio when finding the sum of the geometric progression (ciii).