HSC Marking Feedback 2017

Mathematics

Written Examination

Question 11

(a) Common problems were:

- multiplying the expression by $\frac{\sqrt{5}}{\sqrt{5}}$ or $\frac{\sqrt{5}-1}{\sqrt{5}-1}$
- not using brackets in the numerator, resulting in $2\sqrt{5}+1$
- incorrectly expanding the conjugate product in the denominator.

(b) In better responses, students correctly expanded the expression and then integrated each element.

Common problems were:

- a denominator of 5 rather than 10
- differentiating the given expression.

(c) In better responses, students usually wrote the results for $u, \frac{du}{dx}, v, \frac{dv}{dx}$ before substituting into the quotient rule, or rewrote the function and used the product rule.

Common problems were:

- not differentiating the functions correctly
- differentiating $\sin x$ and x, without applying the quotient rule
- using an incorrect formula for the quotient rule.

(d) In better responses, students usually wrote the results for $u, \frac{du}{dx}, v, \frac{dv}{dx}$ before substituting into the product rule.

Common problems were:

- not differentiating the functions correctly, especially $\ln x$
- incorrectly using the product rule.

(e) (i) Common problems were:

- not correctly using the area of a triangle formula provided on the reference sheet
- using $\frac{1}{2}bh$ with an incorrect perpendicular height or base length

- not evaluating $\sin 30°$.

(e) (ii) Common problems were:

- confusing the terms segment and sector, and their associated area formulae
- substituting $30°$ instead of $\frac{\pi}{6}$ into the area of a segment formula for θ
- subtracting the area of the sector from the area of the triangle, producing a negative result
- using decimal approximations instead of exact values
- expressing $30°$ as $\frac{\pi}{3}$.

(f) In a significant number of responses, students correctly substituted the vertex (2, 1) into the equation of a parabola formula but found it difficult to progress further.

Common problems were:

- incorrectly substituting into the equation of a parabola formula
- assuming $a = 1$ or 4
- misreading the vertex as (1, 2)
- using the x-axis as the directrix
- copying the formula incorrectly from the reference sheet, for example $(x - h)^2 = 4a(y - k)^2$.

(g) Common problems were:

- solving only one equation and ignoring the negative case
- incorrectly solving the negative case
- writing the final answer as an inequality.

(h) In a number of responses, students had difficulty solving $3 - x \geq 0$.

Common problems were:

- solving $3 - x > 0$
- treating the function as a semi-circle and expressing the domain as $-3 \leq x \leq 3$.

Question 12

(a) In most responses, the use of the point-gradient formula was used successfully. Common problems were:

- finding a gradient using an incorrect value for x

- solving $\frac{dy}{dx} = 0$
- using the coefficient of x in the derivative for the gradient.

(b) In most responses, students identified that the volume of solids of revolution involved the use of π, integration over an interval and squaring of the function.

Common problems were:

- rotating about the incorrect axis (y-axis)
- omitting π from the solution
- using incorrect limits and/or substituting limits incorrectly.

(c) In almost all responses, students were able to engage in this part of the question with some success. They were able to use the equations for the arithmetic series provided on the reference sheet and most students proceeded to solve their equations simultaneously.

Common problems were:

- incorrectly copying the information from the examination paper
- making algebraic errors when solving their two simultaneous equations
- not finding the value of the tenth term.

(d) (i) Common problems were:

- not determining the absolute value in the numerator
- finding an incorrect value in the denominator.

(d) (ii) In better responses, students demonstrated that they understood that they needed to find the length CD. Those who knew the area of a trapezium formula used it well.

Common problems were:

- simplifying surds incorrectly
- using incorrect area formulae.

(e) (i) Most responses to this part were correct.

(e) (ii) In most responses, students were able to obtain the correct answer using the complement of P(all odd). Some students added all relevant probabilities.

(e) (iii) In some responses, students used addition rather than multiplying the two probabilities.

(e) (iv) In a small number of responses, students incorrectly cubed their result from (e)(iii).

Question 13

(a) In better responses, students were able to use the cosine rule, expand the quadratic expressions correctly, collect the like terms and solve the resulting quadratic equation.

Common problems included:

- not copying the cosine rule formula correctly from the reference sheet
- incorrectly substituting into the cosine rule
- incorrectly expanding $-2(x+4)(x-4)$
- not evaluating $\cos 60°$
- finding an incorrect value for $\cos 60°$ of $\frac{\sqrt{3}}{2}$ or $\frac{1}{\sqrt{2}}$
- using $\cos 30°$ or $\sin 60°$.

(b) (i) Common problems included:

- factorising incorrectly which led to incorrect x values for their stationary points
- using the first or the second derivative test to determine the nature of a stationary point without valid justification
- finding points of inflexion
- incorrectly substituting stationary points into the derived function to find y-values.

(b) (ii) Common problems graphing the curve included:

- plotting points incorrectly
- changing the nature of their stationary points without valid justification when their working in (b)(i) did not combine to make a cubic curve
- sketching a curve with different features than those found in (b)(i).

(b) (iii) In better responses, students used their graph from (b)(ii) were quite successful. Those who solved $\frac{dy}{dx} > 0$ algebraically often struggled to solve the inequality. Common problems included:

- incorrectly using their graph from (b)(ii)
- writing $-2 > x > 1$
- solving a different inequality.

(c) Common problems included:

- making errors when using logs to find the value of t
- not recognising the need to solve for t and only finding values for m

- stating $t^{\frac{1}{3}} = -3$ has no real solution
- considering t as time and using this reason to exclude negative values.

(d) In better responses, students recognised $\frac{dV}{dt}$ in the form $\frac{f'(t)}{f(t)}$ and integrated to find $V = \ln(1 + t^2)$.

Common problems were:

- not integrating correctly
- not evaluating the constant of integration
- differentiating using the quotient rule
- incorrectly finding the volume by substituting $t = 10$ directly into $\frac{dV}{dt}$.

Question 14

(a) Common problems were:

- sketching a sine curve with period π and amplitude 3, but not moved up 4 units
- sketching the correct graph for $0 \leq x \leq \pi$
- poor labeling of axes and scale.

(b) (i) A common problem was incorrectly stating $\int \cos x \, dx = -\sin x$.

(b) (ii) Common problems were:

- not correctly evaluating $\frac{b-a}{6}$
- incorrectly substituting into Simpson's rule
- using the trapezoidal rule
- finding incorrect exact values for $\cos\frac{\pi}{6}$ and $\cos\frac{\pi}{3}$.

(b) (iii) In better responses, students equated their answers from parts (i) and (ii).

Common problems were:

- not linking the results of (b) (i) and (b) (ii)
- incorrectly rearranging their expression in terms of π
- equating (b)(i) and (b)(ii) but then changing the numbers to generate the answer provided on the examination paper.

(c) (i) In most responses, students were able to differentiate $C(t)$ and verify the statement.

(c) (ii) In most responses, students were able to successfully use half-life to find a value for t.

A common problem was leaving C and A in the equation and not solving for k.

(c) (iii) In better responses, students obtained the correct exponential equation usually obtained the correct answer.

A common problem was using $0.1A$ instead of $0.9A$.

(d) Common problems were:

- making algebraic errors when subtracting the functions
- finding an incorrect primitive
- substituting into k instead of x in the primitive function when evaluating the limits.

Question 15

(a) In better responses, students used clear logic and provided geometric reasons for each step.

Common problems were:

- referring to an angle using only its vertex when there is more than one angle with that vertex
- insufficient setting out with little or no reasoning
- incorrectly naming a pair of parallel lines in a justification
- incorrectly using the terms corresponding, alternate and co-interior angles to name angle pairs
- making algebraic errors when solving equations, especially those involving fractions
- not copying the diagram into their booklet and then introducing a new variable without explanation
- extrapolating the figure to a decagon without explanation and without reference to regular decagon.

(b) (i) In most responses, students were able to find the correct expression for M_1 and many correctly showed how to obtain the expression for M_2.

Common problems were:

- attempting to work backwards from the given expression for M_2
- omitting brackets or not closing brackets
- not including a deposit of $\$X$ at the start of each month
- writing the given statement for M_2 without any working
- not checking their answer for (b)(i) was the same as the one required before continuing onto (b)(ii).

(b) (ii) Common problems were:

- not using the appropriate value for n (48 months)
- using incorrect values for a and n for each series
- incorrectly applying the S_n formula for a geometric series.

(c) (i) Common problems were:

- not evaluating the constant of integration
- substituting the given values to prove they satisfy the equation without integrating
- incorrectly integrating e^{-t}
- not correctly evaluating e^0.

(c) (ii) Common problems were:

- not obtaining a quadratic equation after equating v_1 and v_2
- incorrectly factorising and solving the resulting quadratic equation.

(c) (iii) In most responses, students were able to find x_1 and x_2 and equate the two expressions but had difficulty drawing the correct conclusion.

Common problems were:

- not correctly evaluating e^0
- correctly integrating v_1 and v_2 and omitting the constant of integration
- dividing both sides of their cubic equation by t without acknowledging that $t = 0$ was a solution outside the required domain
- not justifying that the particles do not meet for $t > 0$.

Question 16

(a) (i) In some responses, students found it difficult to respond to the direction "Show". Some provided an answer which predominately included the result provided in the question without any clear explanation. Almost all responses used Pythagoras' theorem. However, not all referred to the sides of the given triangles but instead used the generic $a^2 + b^2 = c^2$.

(a) (ii) In some responses, students recognised the connection between $\frac{dL}{dx} = 0$ and $\sin\alpha = \sin\beta$ and were able to set their work out in clear logical steps. In better responses, students expanded $(9 - x)^2$ and then differentiated, or identified ratios for $\sin\alpha$ and $\sin\beta$.

Common problems were:

- differentiating incorrectly
- integrating instead of differentiating
- not recognising the relationship between $\frac{dL}{dx} = 0$ and $\sin\alpha = \sin\beta$
- incorrectly expanding $49 + (9 - x)^2$

(a) (iii) In better responses, students recognised if $\sin\alpha = \sin\beta$ the triangles were similar, or if $\sin\alpha = \sin\beta$ then $\tan\alpha = \tan\beta$ (α and β acute). In most responses, students tried to solve $\frac{x}{\sqrt{x^2+25}} = \frac{9-x}{\sqrt{49+(9-x)^2}}$ and found the algebraic manipulation required to be challenging.

Common problems were:

- making algebraic errors when solving the equation
- using $\frac{x}{5} = \frac{9-x}{7}$ without explanation.

(a) (iv) In better responses, students responded correctly to the direction "Explain" by demonstrating a deeper engagement. Using a table to test x values above and below $x = \frac{15}{4}$ and explain why their x value gave a minimum proved to be the easier method. Students who attempted to find $\frac{d^2L}{dx^2}$ and substitute their value for x found that method difficult.

(b) In most responses, students were able to achieve $2 = \frac{a}{1-r}$ and appreciated that $|r| < 1$ but did not link both statements to achieve the correct result. In better responses, students stated incorrectly that $|r| \leq 1$ or $0 < r < 1$.

Common problems were:

- incorrect algebraic manipulation when making r the subject of $2 = \frac{a}{1-r}$
- solving inequations incorrectly.

(c) (i) In better responses, students used the property of equal intercepts and parallel lines or proved triangles BDM and BEC were similar triangles, to show $BD = DE$.

Common problems were:

- attempting to prove ΔACE is isosceles without using the information provided
- stating that D is the midpoint of BE without justification.

(c) (ii) Common problems were:

- proving extra triangles were similar (using ΔBDM, ΔBAF and ΔBEC) and hence manipulated many equal angles to achieve the correct result
- naming angles incorrectly
- providing incorrect geometric reasons.